Mathematics 5

Implementation Draft
July 2014

Mathematics 5, Implementation Draft

Cataloguing-in-Publication Data

Acknowledgements

The Nova Scotia Department of Education and Early Childhood Education wishes to express its gratitude to the following organizations for granting permission to adapt their mathematics curriculum in the development of this guide.

Manitoba Education

New Brunswick Department of Education

Newfoundland and Labrador Department of Education

The Western and Northern Canadian Protocol (WNCP) for Collaboration in Education

We also gratefully acknowledge the contributions of the following individuals toward the development of the Nova Scotia Mathematics 5 curriculum.

Gaston Comeau
South Shore Regional School Board

Bob Crane
Mi'kmaw Kina'matnewey

Robin Harris
Halifax Regional School Board

Darlene MacKeen Hudson
Chignecto-Central Regional School Board

Mark MacLeod
South Shore Regional School Board

Rebecca McDonald
Chignecto-Central Regional School Board

Sonya O'Sullivan
Halifax Regional School Board

Novadawn Oulton
Annapolis Valley Regional School Board

Mark Pettipas
Strait Regional School Board

Fred Sullivan
Strait Regional School Board

Marlene Urquhart
Cape Breton-Victoria Regional School Board

Tom Willis
Tri-County Regional School Board

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Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K–9 Mathematics* (2006) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework (WNCP 2006) was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the province of Nova Scotia. It should also enable easier transfer for students moving within the province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Assessment

Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes

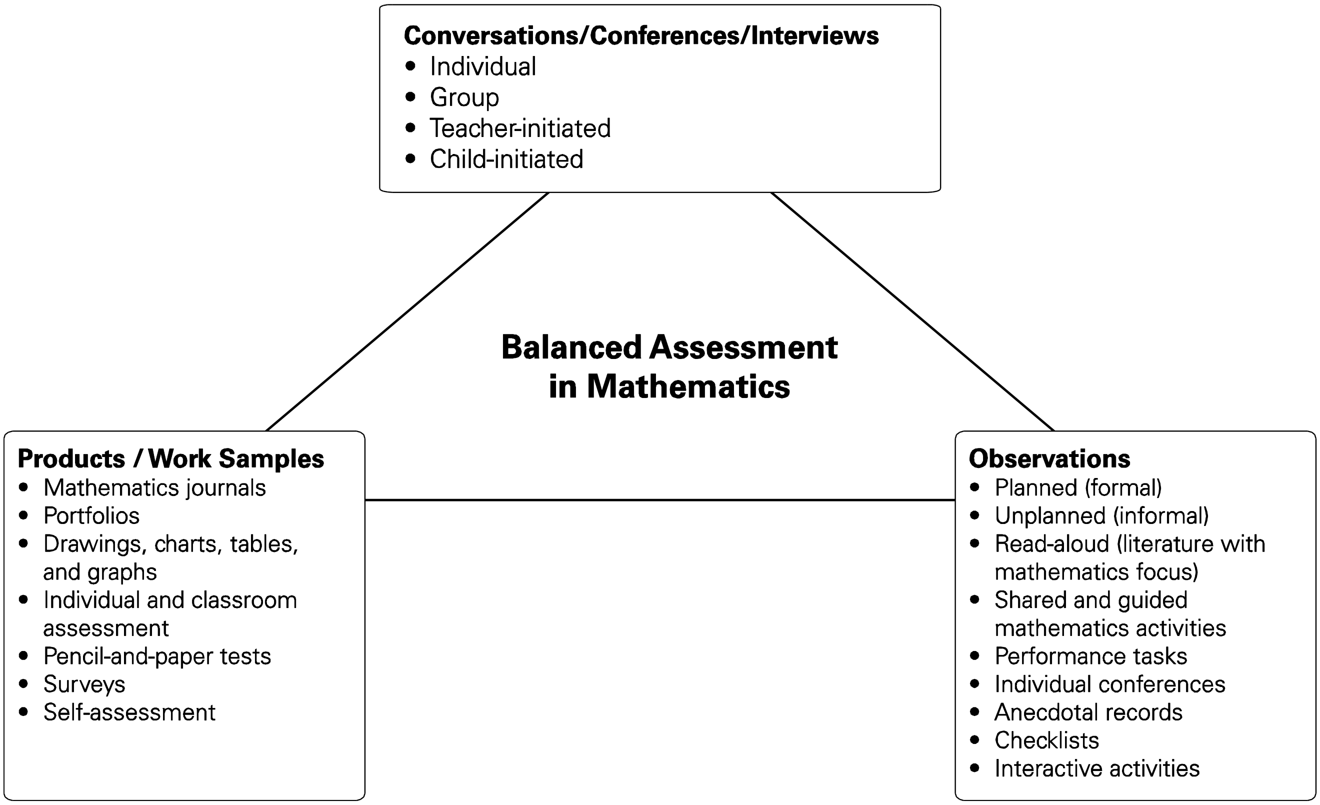
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

Assessment of student learning should

- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction



Time to Learn for Mathematics

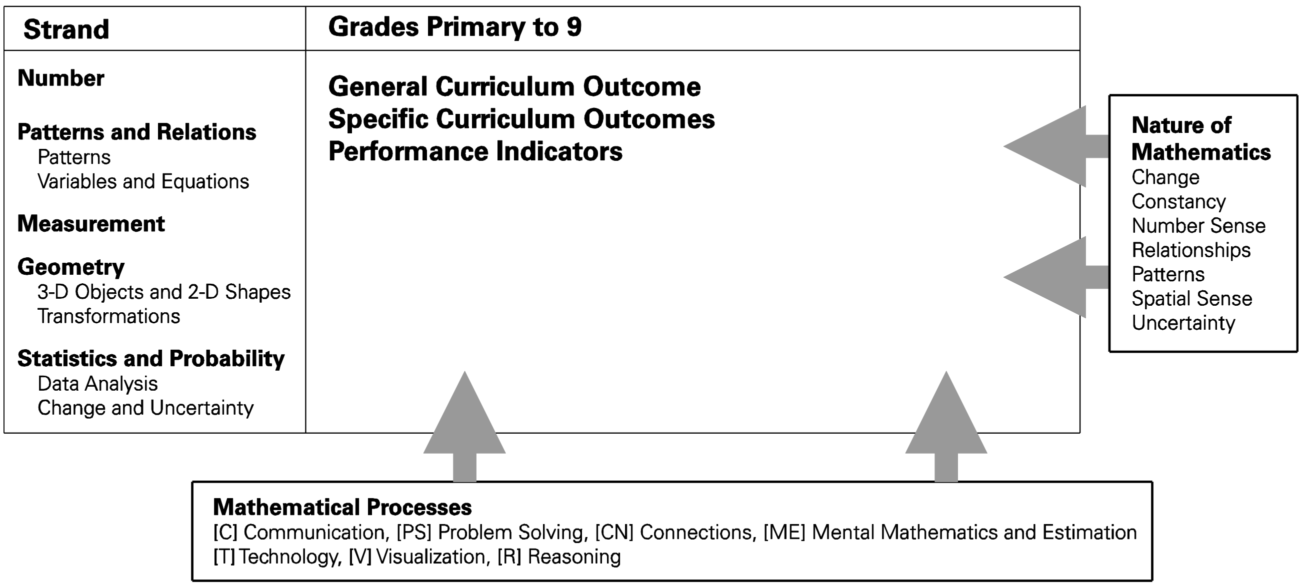
The *Time to Learn Strategy, Guidelines for Instructional Time: Grades Primary*–6 (Nova Scotia Department of Education 2002) includes time for mathematics instruction in the "Required Each Day" section. In order to support a constructivist approach to teaching through problem solving, it is highly recommended that the 45 minutes required daily in grades primary–2 and the 60 minutes required daily for grades 3–6 mathematics instruction be provided in an uninterrupted block of time.

Time to Learn guidelines can be found at
www.ednet.ns.ca/files/ps-policies/semestering.pdf
www.ednet.ns.ca/files/ps-policies/instructional_time_guidelines_p-6.pdf

Outcomes

Conceptual Framework for Mathematics Primary–9

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



(Adapted with permission from Western and Northern Canadian Protocol, *The Common Curriculum Framework for K–9 Mathematics*, p. 5. All rights reserved.)

Structure of the Mathematics Curriculum

Strands

The learning outcomes in the Nova Scotia Framework are organized into five strands across grades primary to 9.

- Number (N)
- Patterns and Relations (PR)
- Measurement (M)
- Geometry (G)
- Statistics and Probability (SP)

General Curriculum Outcomes (GCO)

Some strands are further subdivided into sub-strands. There is one general outcome (GCO) per sub-strand. GCOs are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

NUMBER (N)

GCO: Students will be expected to demonstrate number sense.

PATTERNS AND RELATIONS (PR)

Patterns
GCO: Students will be expected to use patterns to describe the world and solve problems.

Variables and Equations
GCO: Students will be expected to represent algebraic expressions in multiple ways.

MEASUREMENT (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

GEOMETRY (G)

3-D Objects and 2-D Shapes
GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Transformations
GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

STATISTICS AND PROBABILITY (SP)

Data Analysis
GCO: Students will be expected to collect, display, and analyze data to solve problems.

Chance and Uncertainty
GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes (SCOs) are statements that identify the specific conceptual understanding, related skills, and knowledge students are expected to attain by the end of a given grade.

Performance indicators are statements that identify specific expectations of the depth, breadth, and expectations for the outcome. Teachers use these statements to determine whether students have achieved the corresponding specific curriculum outcome.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

NUMBER (N)

N01 Students will be expected to represent and partition whole numbers to 1 000 000. [C, CN, V, T]

Performance Indicators

N01.01 Read a given numeral without using the word "and."
N01.02 Record numerals for numbers expressed orally, concretely, pictorially, or symbolically as expressions, using proper spacing without commas.
N01.03 Describe the pattern of adjacent place positions moving from right to left.
N01.04 Explain the meaning of each digit in a given numeral.
N01.05 Provide examples of large numbers used in print or electronic media.
N01.06 Express a given numeral in expanded notation.
N01.07 Write the numeral represented by a given expanded notation.
N01.08 Compare and order numbers to 1 000 000 in a variety of ways.
N01.09 Represent a given numeral, 0 to 1 000 000, using a place-value chart.
N01.10 Represent a given number, 0 to 1 000 000, in a variety of ways, and explain how they are equivalent.
N01.11 Represent a given number, 0 to 1 000 000, using expressions.
N01.12 Read and write given numerals, 0 to 1 000 000, in words.

N02 Students will be expected to use estimation strategies, including front-end, front-end adjusted, rounding, and compatible numbers in problem-solving contexts. [C, CN, ME, PS, R, V]

Performance Indicators

N02.01 Provide a context for when estimation is used to make predictions, check the reasonableness of an answer, and determine approximate answers.
N02.02 Describe contexts in which overestimating is important.
N02.03 Determine the approximate solution to a given problem not requiring an exact answer.
N02.04 Estimate a sum, a difference, a product, or a quotient using an appropriate strategy.
N02.05 Select and explain an estimation strategy for a given problem.

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N02.03 Determine the approximate solution to a given problem not requiring an exact answer.
N02.04 Estimate a sum, a difference, a product, or a quotient using an appropriate strategy.
N02.05 Select and explain an estimation strategy for a given problem.

N03 Students will be expected to describe and apply mental mathematics strategies and number properties to recall, with fluency, answers for basic multiplication facts to 81 and related division facts. [C, CN, ME, R, V]

Performance Indicators

N03.01 Describe the mental mathematics strategy used to determine basic multiplication or division facts.
N03.02 Explain why multiplying by 0 produces a product of 0 (zero property of multiplication).
N03.03 Explain why division by 0 is not possible or is undefined (e.g., 8 ÷ 0).
N03.04 Quickly recall multiplication facts up to 9 × 9 and related division facts.

N04 Students will be expected to apply mental mathematics strategies for multiplication, including

- multiplying by multiples of 10, 100, and 1000
- halving and doubling
- using the distributive property

[C, ME, R]

Performance Indicators

N04.01 Determine the products when one factor is a multiple of 10, 100, or 1000.
N04.02 Apply halving and doubling when determining a given product (e.g., 32 × 5 is the same as 16 × 10).
N04.03 Apply the distributive property to determine a given product that involves multiplying factors that are close to multiples of 10 (e.g., 98 × 7 = (100 × 7) – (2 × 7)).

N05 Students will be expected to demonstrate, with and without concrete materials, an understanding of multiplication (two-digit by two-digit) to solve problems. [C, CN, PS, V]

Performance Indicators

N05.01 Model the multiplication of two two-digit factors, using concrete and visual representations of the area model, and record the process symbolically.
N05.02 Illustrate partial products in expanded notation for both factors (e.g., For 36 × 42, determine the partial products for (30 + 6) × (40 + 2).).
N05.03 Represent both two-digit factors in expanded notation to illustrate the distributive property (e.g., To determine the partial products of 36 × 42, record (30 + 6) × (40 + 2) = 30 × 40 + 30 × 2 + 6 × 40 + 6 × 2 = 1200 + 60 + 240 + 12 = 1512.).
N05.04 Describe a solution procedure for determining the product of two given two-digit factors, using a pictorial representation such as an area model.
N05.05 Solve a given multiplication problem in context, using personal strategies, and record the process.
N05.06 Create and solve multiplication story problems, and record the process symbolically.
N05.07 Determine the product of two given numbers using a personal strategy and record the process symbolically.

N08.04 Express a given tenth as an equivalent hundredth and thousandth.
N08.05 Express a given hundredth as an equivalent thousandth.
N08.06 Explain the value of each digit in a given decimal.

N09 Students will be expected to relate decimals to fractions and fractions to decimals (to thousandths). [CN, R, V]

Performance Indicators

N09.01 Express, orally and symbolically, a given fraction with a denominator of 10, 100, or 1000 as a decimal.
N09.02 Read decimals as fractions (e.g., 0.45 is read as zero and forty-five hundredths).
N09.03 Express, orally and symbolically, a given decimal in fraction form.
N09.04 Represent the fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$ as decimals using base-ten blocks, grids, and number lines.
N09.05 Express a given pictorial or concrete representation as a fraction or decimal (e.g., 250 shaded squares on a thousandth grid can be expressed as 0.250 or $\frac{250}{1000}$).

N10 Students will be expected to compare and order decimals (to thousandths) by using benchmarks, place value, and equivalent decimals. [CN, R, V]

Performance Indicators

N10.01 Compare and order a given set of decimals by placing them on a number line that contains the benchmarks 0.0, 0.5, and 1.0.
N10.02 Compare and order a given set of decimals including only tenths using place value.
N10.03 Compare and order a given set of decimals including only hundredths using place value.
N10.04 Compare and order a given set of decimals including only thousandths using place value.
N10.05 Explain what is the same and what is different about 0.2, 0.20, and 0.200.
N10.06 Compare and order a given set of decimals, including tenths, hundredths, and thousandths, using equivalent decimals.

N11 Students will be expected to demonstrate an understanding of addition and subtraction of decimals (limited to thousandths). [C, CN, ME, PS, R, V]

Performance Indicators

N11.01 Predict sums and differences of decimals using estimation strategies.
N11.02 Use estimation to correct errors of decimal point placements in sums and differences without using paper and pencil.
N11.03 Explain why keeping track of place-value positions is important when adding and subtracting decimals.
N11.04 Solve problems that involve addition and subtraction of decimals, limited to thousandths, using personal strategies.

PATTERNS AND RELATIONS (PR)

PR01 Students will be expected to determine the pattern rule to make predictions about subsequent terms. [C, CN, PS, R, V]

Performance Indicators

PR01.01 Extend a given increasing or decreasing pattern, with and without concrete materials, and explain how each term differs from the preceding one.
PR01.02 Describe, orally or in written form, a given pattern using mathematical language such as **one more**, **one less**, or **five more**.
PR01.03 Write a mathematical expression to represent a given pattern, such as $r + 1$, $r - 1$, $r + 5$.
PR01.04 Describe the relationship in a given table or chart using a mathematical expression.
PR01.05 Determine and explain why a given number is or is not the next term in a pattern.
PR01.06 Predict subsequent terms in a given pattern.
PR01.07 Solve a given problem by using a pattern rule to determine subsequent terms.
PR01.08 Represent a given pattern visually to verify predictions.

PR02 Students will be expected to solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions. [C, CN, PS, R]

Performance Indicators

PR02.01 Explain the purpose of the letter variable in a given addition, subtraction, multiplication, or division equation with one unknown (e.g., $36 \div n = 6$).
PR02.02 Express a given pictorial or concrete representation of an equation in symbolic form.
PR02.03 Express a given problem as an equation where the unknown is represented by a letter variable.
PR02.04 Create a problem for a given equation with one unknown.
PR02.05 Solve a given single-variable equation with the unknown in any of the terms (e.g., $n + 2 = 5$, $4 + a = 7$, $6 = r - 2$, $10 = 2c$, $15 \div r = 3$).
PR02.06 Identify the unknown in a problem; represent the problem with an equation; and solve the problem concretely, pictorially, or symbolically.

MEASUREMENT (M)

M01 Students will be expected to design and construct different rectangles, given a perimeter or an area or both (whole numbers), and make generalizations. [C, CN, PS, R, V]

Performance Indicators

M01.01 Draw two or more rectangles for a given perimeter in a problem-solving context.
M01.02 Draw two or more rectangles for a given area in a problem-solving context.
M01.03 Determine the shape that will result in the greatest area for any given perimeter.
M01.04 Determine the shape that will result in the least area for any given perimeter.
M01.05 Provide a real-life context for when it is important to consider the relationship between area and perimeter.

M02 Students will be expected to demonstrate an understanding of measuring length (mm) by

- selecting and justifying referents for the unit millimetre (mm)
- modelling and describing the relationship between millimetre (mm) and centimetre (cm) units, and between millimetre (mm) and metre (m) units

[C, CN, ME, PS, R, V]

Performance Indicators

M02.01 Provide a referent for one millimetre, and explain the choice.
M02.02 Provide a referent for one centimetre, and explain the choice.
M02.03 Provide a referent for one metre, and explain the choice.
M02.04 Show that 10 millimetres is equivalent to one centimetre, using concrete materials.
M02.05 Show that 1000 millimetres is equivalent to one metre, using concrete materials.
M02.06 Provide examples of instances where millimetres are used as the unit of measure.
M02.07 Estimate and measure length in millimetres, centimetres, and metres.

M03 Students will be expected to demonstrate an understanding of volume by

- selecting and justifying referents for cubic centimetre (cm^3) or cubic metre (m^3) units
- estimating volume using referents for cubic centimetre (cm^3) or cubic metre (m^3)
- measuring and recording volume (cm^3 or m^3)
- constructing rectangular prisms for a given volume

[C, CN, ME, PS, R, V]

Performance Indicators

M03.01 Identify and explain why the cube is the most efficient unit for measuring volume.
M03.02 Provide a referent for a cubic centimetre, and explain the choice.
M03.03 Provide a referent for a cubic metre, and explain the choice.
M03.04 Determine which standard cubic unit is represented by a given referent.
M03.05 Estimate the volume of a given 3-D object using personal referents.
M03.06 Determine the volume of a given 3-D object using manipulatives, and explain the strategy.
M03.07 Construct a rectangular prism for a given volume.
M03.08 Construct more than one rectangular prism for a given volume.

M04 Students will be expected to demonstrate an understanding of capacity by

- describing the relationship between millilitre (mL) and litre (L) units
- selecting and justifying referents for millilitre (mL) and litre (L) units
- estimating capacity using referents for millilitre (mL) and litre (L)
- measuring and recording capacity (mL or L)

[C, CN, ME, PS, R, V]

Performance Indicators

M04.01 Demonstrate that 1000 millilitres is equivalent to one litre by filling a one-litre container using a combination of smaller containers.
M04.02 Provide a referent for one litre, and explain the choice.
M04.03 Provide a referent for one millilitre, and explain the choice.
M04.04 Determine the capacity unit of a given referent.
M04.05 Estimate the capacity of a given container using personal referents.
M04.06 Determine the capacity of a given container using materials that take the shape of the inside of the container (e.g., a liquid, rice, sand, beads), and explain the strategy.

GEOMETRY (G)

G01 Students will be expected to describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are parallel, intersecting, perpendicular, vertical, and horizontal. [C, CN, R, T, V]

Performance Indicators

G01.01 Identify parallel, intersecting, perpendicular, vertical, and horizontal edges and faces on 3-D objects.
G01.02 Identify parallel, intersecting, perpendicular, vertical, and horizontal sides on 2-D shapes.
G01.03 Provide examples from the environment that show parallel, intersecting, perpendicular, vertical, and horizontal line segments.
G01.04 Find examples of edges, faces, and sides that are parallel, intersecting, perpendicular, vertical, and horizontal in print and electronic media, such as newspapers, magazines, and the Internet.
G01.05 Draw 2-D shapes that have sides that are parallel, intersecting, perpendicular, vertical, or horizontal.
G01.06 Build 3-D objects that have edges and faces that are parallel, intersecting, perpendicular, vertical, or horizontal.
G01.07 Describe the faces and edges of a given 3-D object using terms such as **parallel**, **intersecting**, **perpendicular**, **vertical**, or **horizontal**.
G01.08 Describe the sides of a given 2-D shape using terms such as **parallel**, **intersecting**, **perpendicular**, **vertical**, or **horizontal**.

G02 Students will be expected to name, identify, and sort quadrilaterals, including rectangles, squares, trapezoids, parallelograms, and rhombi, according to their attributes. [C, R, V]

Performance Indicators

G02.01 Identify and describe the characteristics of a pre-sorted set of quadrilaterals.
G02.02 Sort a given set of quadrilaterals, and explain the sorting rule.
G02.03 Sort a given set of quadrilaterals according to the lengths of the sides.
G02.04 Sort a given set of quadrilaterals according to whether or not opposite sides are parallel.
G02.05 Sort a set of quadrilaterals based on properties such as diagonals are congruent, diagonals bisect each other, and opposite angles are equal.
G02.06 Name and classify quadrilaterals according to their attributes.

G03 Students will be expected to perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image. [C, CN, T, V]

Performance Indicators

G03.01 Translate a given 2-D shape horizontally, vertically, or diagonally, draw the image, and describe the position and orientation of the image.
G03.02 Rotate a given 2-D shape about a vertex, draw the image, and describe the position and orientation of the image.
G03.03 Reflect a given 2-D shape in a line of reflection, draw the image, and describe the position and orientation of the image.
G03.04 Perform a transformation of a given 2-D shape by following instructions.

G03.05 Draw a 2-D shape, translate the shape, and record the translation by describing the direction and magnitude of the movement.
G03.06 Draw a 2-D shape, rotate the shape about a vertex, and describe the direction of the turn (clockwise or counter-clockwise) and the fraction of the turn (limited to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or full turn).
G03.07 Draw a 2-D shape, reflect the shape, and identify the line of reflection and the distance of the image from the line of reflection.
G03.08 Predict the result of a single transformation of a 2-D shape and verify the prediction.

G04 Students will be expected to identify and describe a single transformation, including a translation, rotation, and reflection of 2-D shapes. [C, T, V]

Performance Indicators

G04.01 Provide an example of a translation, rotation, and reflection.
G04.02 Identify a given single transformation as a translation, rotation, or reflection.
G04.03 Describe a given rotation about a point of rotation by the direction of the turn (clockwise or counter-clockwise).
G04.04 Describe a given reflection by identifying the line of reflection and the distance of the image from the line of reflection.
G04.05 Describe a given translation by identifying the direction and magnitude of the movement.
G04.06 Identify transformations found in everyday pictures, art, or the environment.

G05 Students will be expected to identify right angles. [ME, V]

Performance Indicators

G05.01 Provide examples of right angles in the environment.
G05.02 Sketch right angles without the use of a protractor.
G05.03 Label a right angle, using a symbol.
G05.04 Identify angles greater than or less than a right angle.

Statistics and Probability (SP)

SP01 Students will be expected to differentiate between first-hand and second-hand data. [C, R, T, V]

Performance Indicators

SP01.01 Explain the difference between first-hand and second-hand data.
SP01.02 Formulate a question that can best be answered using first-hand data and explain why.
SP01.03 Formulate a question that can best be answered using second-hand data and explain why.
SP01.04 Find examples of second-hand data in print and electronic media, such as newspapers, magazines, and the Internet.

SP02 Students will be expected to construct and interpret double bar graphs to draw conclusions. [C, PS, R, T, V]

Performance Indicators

SP02.01 Determine the attributes (title, axes, intervals, and legend) of double bar graphs by comparing a given set of double bar graphs.
SP02.02 Represent a given set of data by creating a double bar graph, label the title and axes, and create a legend without the use of technology.
SP02.03 Draw conclusions from a given double bar graph to answer questions.
SP02.04 Identify examples of double bar graphs used in a variety of print and electronic media, such as newspapers, magazines, and the Internet.
SP02.05 Solve a given problem by constructing and interpreting a double bar graph.

SP03 Students will be expected to describe the likelihood of a single outcome occurring, using words such as **impossible**, **possible**, and **certain**. [C, CN, PS, R]

Performance Indicators

SP03.01 Identify examples of events from personal contexts that are impossible, possible, or certain.
SP03.02 Classify the likelihood of a single outcome occurring in a probability experiment as impossible, possible, or certain.
SP03.03 Design and conduct a probability experiment in which the likelihood of a single outcome occurring is impossible, possible, or certain.
SP03.04 Conduct a given probability experiment a number of times, record the outcomes, and explain the results.

SP04 Students will be expected to compare the likelihood of two possible outcomes occurring, using words such as **less likely**, **equally likely**, or **more likely**. [C, CN, PS, R]

Performance Indicators

SP04.01 Identify outcomes from a given probability experiment that are less likely, equally likely, or more likely to occur than other outcomes.
SP04.02 Design and conduct a probability experiment in which one outcome is less likely to occur than the other outcome.
SP04.03 Design and conduct a probability experiment in which one outcome is equally likely to occur as the other outcome.
SP04.04 Design and conduct a probability experiment in which one outcome is more likely to occur than the other outcome.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to
- communicate in order to learn and express their understanding of mathematics (Communication [C])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- select and use technologies as tools for learning and solving problems (Technology [T])
- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])
- develop mathematical reasoning (Reasoning [R])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific curriculum outcome within the strands.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to clarify their thinking and provides teachers with insight into students' interpretations of mathematical meanings and ideas.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, How would you ...? or How could you ...? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

When students are exposed to a wide variety of problems in all areas of mathematics, they explore various methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problem.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

“Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding. … *Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*.” (Caine and Caine 1991, 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

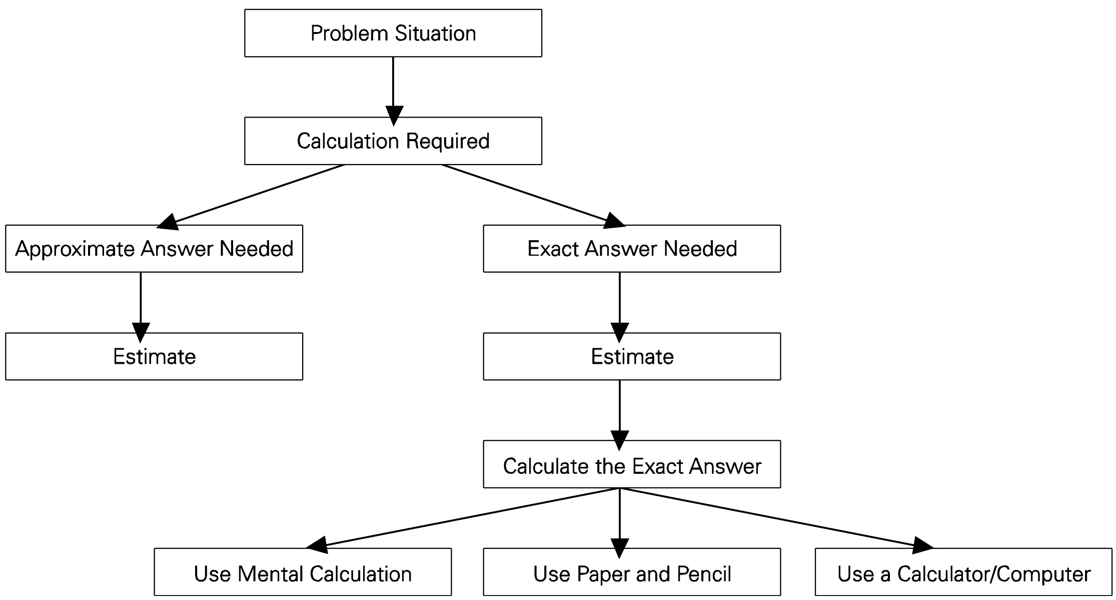
Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. “Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math.” (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving.” (Rubenstein 2001) Mental mathematics “provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers.” (Hope 1988, v)

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated below.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems. Information and communication technology best improves learning when it is accessible, flexible, responsive, participatory, and integrated thoroughly into all public school programs.

Technology can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate

- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. Calculators do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in grades primary–3 to enrich learning, it is expected that students will achieve all outcomes without the use of technology. The Integration of Information and Communication Technology within the Classroom, 2012 (P–6) can be found online in several locations, including http://lrt.ednet.ns.ca.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world." (Armstrong 1999). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. These mental images are needed to develop concepts and understand procedures. Images and explanations help students clarify their understanding of mathematical ideas in all strands.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, 184).

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state**, and **symmetry** (AAAS–Benchmarks 1993, 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education 2000, 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands, and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with an understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example,

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to achieve during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The footer of the document shows the name of the course, and the strand name is presented in the header. When a specific curriculum outcome (SCO) is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also for each SCO, there is background information, assessment strategies, suggested instructional strategies, suggested models and manipulatives, mathematical language, and a section for resources and notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

SCO

Mathematical Processes
[C] Communication **[PS]** Problem Solving **[CN]** Connections
[ME] Mental Mathematics and Estimation
[T] Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Describes observable indicators of whether students have achieved the specific outcome.

Scope and Sequence

Previous grade or course SCOs	Current grade SCO	Following grade or course SCOs

Background

Describes the "big ideas" to be learned and how they relate to work in previous grade and work in subsequent courses.

Additional Information

A reference to Appendix A, which contains further elaborations for the performance indicators.

Assessment, Teaching, and Learning

Assessment Strategies

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Sample tasks that can be used to determine students' prior knowledge.

Whole-Class/Group/Individual Assessment Tasks

Some suggestions for specific activities and questions that can be used for both instruction and assessment

Follow-up on Assessment

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcome and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Suggested strategies for planning daily lessons.

Suggested Learning Tasks

Suggestions for general approaches and strategies suggested for teaching this outcome.

Guiding Questions
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Suggested Models and Manipulatives

Mathematical Language

Teacher and student mathematical language associated with the respective outcome.

Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

"Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge." (National Council of Teachers of Mathematics 2000, 20).

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Goals for Mathematics Education

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals or assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners

"No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn.*" (Hume 2011, 6)

Student engagement is at the core of learning. Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences that are both age and developmentally appropriate.

This curriculum is designed to provide learning opportunities that are equitable, accessible, and inclusive of the many facets of diversity represented in today's classrooms. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, persist in challenging situations, and apply reflective practices.

Supportive Learning Environments

A supportive and positive learning environment has a profound effect on students' learning. Students need to feel physically, socially, emotionally, and culturally safe in order to take risks with their learning. In classrooms where students feel a sense of belonging, see their teachers' passion for learning and teaching, are encouraged to actively participate, and are challenged appropriately, they are more likely to be successful.

Teachers recognize that not all students progress at the same pace nor are they equally positioned in terms of their prior knowledge of particular concepts, skills, and learning outcomes. Teachers are able to create more equitable access to learning when

- instruction and assessment are flexible and offer multiple means of representation
- students have options to engage in learning through multiple ways
- students can express their knowledge, skills, and understanding in multiple ways

(Hall, Meyer, and Rose 2012)

In a supportive learning environment, teachers plan learning experiences that support *each* student's ability to achieve curriculum outcomes. Teachers use a variety of effective instructional approaches that help students to succeed, such as

- providing a range of learning opportunities that build on individual strengths and prior knowledge
- providing all students with equitable access to appropriate learning strategies, resources, and technology
- involving students in the creation of criteria for assessment and evaluation
- engaging and challenging students through inquiry-based practices
- verbalizing their own thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class learning experiences
- scaffolding instruction and assignments as needed and giving frequent and meaningful descriptive feedback throughout the learning process
- integrating "blended learning" opportunities by including an online environment that extends learning beyond the physical classroom
- encouraging students to take time and to persevere, when appropriate, in order to achieve a particular learning outcome

Multiple Ways of Learning

"Advances in neuroscience and education research over the past 40 years have reshaped our understanding of the learning brain. One of the clearest and most important revelations stemming from brain research is that there is no such thing as a 'regular student.'" (Hall, Meyer, and Rose 2012, 2) Teachers who know their students well are aware of students' individual learning differences and use this understanding to inform instruction and assessment decisions.

The ways in which students make sense of and demonstrate learning vary widely. Individual students tend to have a natural inclination toward one or a few learning styles. Teachers are often able to detect learning strengths and styles through observation and through conversation with students. Teachers can also get a sense of learning styles through an awareness of students' personal interests and talents. Instruction and assessment practices that are designed to account for multiple learning styles create greater opportunities for all students to succeed.

While multiple learning styles are addressed in the classroom, the three most commonly identified are:

- auditory (such as listening to teacher-modelled think-aloud strategies or participating in peer discussion)
- kinesthetic (such as examining artifacts or problem-solving using tools or manipulatives)
- visual (such as reading print and visual texts or viewing video clips)

For additional information, refer to *Frames of Mind: The Theory of Multiple Intelligences* (Gardner 2007) and *How to Differentiate Instruction in Mixed-Ability Classrooms* (Tomlinson 2001).

A Gender-Inclusive Curriculum and Classroom

It is important that the curriculum and classroom climate respect the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they

- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language, inclusive practices, and respectful listening in their interactions with students
- identify and openly address societal biases with respect to gender and sexual identity

Valuing Diversity: Teaching with Cultural Proficiency

“Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students’ engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995).” (Herzig 2005)

Teachers appreciate that students have diverse life and cultural experiences and that individual students bring different prior knowledge to their learning. Teachers can build upon their knowledge of their students as individuals, value their prior experiences, and respond by using a variety of culturally-proficient instruction and assessment practices in order to make learning more engaging, relevant, and accessible for all students. For additional information, refer to *Racial Equity Policy* (Nova Scotia Department of Education 2002) and *Racial Equity / Cultural Proficiency Framework* (Nova Scotia Department of Education 2011).

Students with Language, Communication, and Learning Challenges

Today’s classrooms include students who have diverse language backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students and by conversing with students and/or their families, teachers gain deeper insights into the student as a learner. Teachers can use this awareness to identify and respond to areas where students may need additional support to achieve their learning goals. For students who are experiencing difficulties, it is important that teachers distinguish between those students for whom curriculum content is challenging and those for whom language-based factors are at the root of apparent academic difficulties. Students who are learning English as an additional language may require individual support, particularly in language-based subject areas, while they become more proficient in their English language skills. Teachers understand that many students who appear to be disengaged may be experiencing difficult life or family circumstances, mental health challenges, or low self-esteem, resulting in a loss of confidence that affects their engagement in learning. A caring, supportive teacher demonstrates belief in the students’ abilities to learn and uses the students' strengths to create small successes that help nurture engagement in learning and provide a sense of hope.

STUDENTS WHO DEMONSTRATE EXCEPTIONAL TALENTS AND GIFTEDNESS

Modern conceptions of giftedness recognize diversity, multiple forms of giftedness, and inclusivity. Some talents are easily observable in the classroom because they are already well developed and students have opportunities to express them in the curricular and extracurricular activities commonly offered in schools. Other talents only develop if students are exposed to many and various domains and hands-on experiences. Twenty-first century learning supports the thinking that most students are more engaged when learning activities are problem-centred, inquiry-based, and open-ended. Talented and gifted students usually thrive when such learning activities are present. Learning experiences may be enriched by offering a range of activities and resources that require increased cognitive demand and higher-level thinking with different degrees of complexity and abstraction. Teachers can provide further challenges and enhance learning by adjusting the pace of instruction and the breadth and depth of concepts being explored. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in health education, literacy, music, physical education, science, social studies, and visual arts.

Number (N)

GCO: Students will be expected to demonstrate number sense.

SCO N01 Students will be expected to represent and partition whole numbers to 1 000 000. [C, CN, T, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N01.01 Read a given numeral without using the word "and."
N01.02 Record numerals for numbers expressed orally, concretely, pictorially, or symbolically as expressions, using proper spacing without commas.
N01.03 Describe the pattern of adjacent place positions moving from right to left.
N01.04 Explain the meaning of each digit in a given numeral.
N01.05 Provide examples of large numbers used in print or electronic media.
N01.06 Express a given numeral in expanded notation.
N01.07 Write the numeral represented by a given expanded notation.
N01.08 Compare and order numbers to 1 000 000 in a variety of ways.
N01.09 Represent a given numeral, 0 to 1 000 000, using a place-value chart.
N01.10 Represent a given number, 0 to 1 000 000, in a variety of ways, and explain how they are equivalent.
N01.11 Represent a given number, 0 to 1 000 000, using expressions.
N01.12 Read and write given numerals, 0 to 1 000 000, in words.

Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
N01 Students will be expected to represent and partition whole numbers to 10 000.	**N01** Students will be expected to represent and partition whole numbers to 1 000 000.	**N01** Students will be expected to demonstrate an understanding of place value for numbers greater than one million and less than one thousandth.

Background

Most of the work done by students in Mathematics 5 will involve numbers in the tens and hundreds of thousands; however, they are now expected to develop meaning for "one million." For example, students should understand that one more than 999 999 is 1 000 000 or one thousand sets of 1000 is one million.

Students will continue to use whole numbers to perform computations and as they read and interpret data. In order to have a better understanding of large numbers, such as one million, students need opportunities to investigate problems involving these numbers.

Students should have many opportunities to

- **read** numbers several ways (e.g., 879 346 is read "eight hundred seventy-nine thousand three hundred forty-six" but might also be renamed as 87 ten thousands, 9 thousands, 346 ones; 8 hundred thousands, 79 thousands, 34 tens, 6 ones; or 879 thousands, 3 hundreds, 30 tens, 16 ones. The word "and" is reserved for reading the decimal in decimal numbers.)
- **record** numbers (e.g., ask students to **write** eight hundred thousand sixty; a number which is eighty less than one million; as well as write numbers in **standard form** (741 253), as **expressions** (500 000 + 200 000 + 40 000 + 500 + 500 + 200 + 53), and in **expanded notation** (700 000 + 40 000 + 1000 + 200 + 50 + 3). For numbers with more than four digits, spaces are used between groups of three digits instead of commas [e.g., 29 304].)
- **establish** referents for one million, one-half million, one hundred thousand, and so on, in order to develop a sense of larger numbers
- **model** numbers using a variety of concrete and pictorial representations including
  - base-ten blocks (e.g., recognize that 1000 large cubes would represent 1 000 000)
  - money (e.g., How many $100 bills are there in $9,347?)
  - place-value charts

Through these experiences, students will develop flexibility in identifying and representing numbers up to 1 000 000.

It is also important for students to gain an understanding of the relative size (magnitude) of numbers through real-life contexts that are personally meaningful. Students should establish **personal referents** to think about large numbers. **Benchmarks** that students may find helpful are multiples of 100, 1000, 10 000, and 100 000, as well as 250 000, 500 000, and 750 000 (one-fourth or one-quarter, one-half, and three-fourths or three-quarters of one million).

The focus of instruction should be on ensuring students develop a strong sense of number. The development of this outcome should be ongoing through the year.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to use base-ten blocks to model 2016 in three different ways. Invite students to explain their models.
- Ask students, Which of the expressions below represent 360? Ask them to explain their thinking.
  - 200 – 160
  - 380 – 30
  - 400 – 40
  - 300 + 60
  - 100 + 100 + 100 + 50 + 10
  - 260 + 75 + 25
  - 357 + 3
  - 260 + 10

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to record a series of numbers that have been read to them. Ensure students include correct spacing without commas. Invite students to express those same numbers in expanded notation.
- Ask students to explain how one million compares to 1000, 10 000, or 100 000.
- Invite students to record a number that is 10 000 more than a given number (or variations of this such as 1000 less, 20 000 less, or 100 000 more).
- Ask students to use newspapers or catalogues to find items that would have a total cost of $20,000, $100,000, or $1 million.
- Ask students to explain how they know that 100 000 is the same as 100 thousands or that 1 000 000 is the same as 1000 thousands.
- Tell students that you bought a car with 50 hundred dollar bills, 20 thousand dollar bills, 100 ten dollar bills, and 46 loonies. Ask students to determine the cost of the car.
- Ask students to describe when 50 000, 500 000, or 1 000 000 of something might be a big amount. A small amount.
- Provide students with a set of numbers (up to seven-digits) written in words and ask students to write the numbers in standard form (using correct spacing and no commas).
- Ask students to insert two zeros anywhere in the number 3759 to form a new six-digit number. Ask them to read the new number and to explain how the place value of each digit has changed.
- Provide students with a set of numbers written in expanded form and ask students to write them in standard form.
- Provide students with a set of numbers written in standard form and invite students to write them in expanded notation.

- Ask students to explain how the value of the digit "1" changed in each of the following numbers:
  - 2681
  - 1 000 000
  - 918 702
  - 103 557
- Ask students to record a six-digit number of their choice. Ask them to read the number. Then, ask them to record the number that is 10 000 more than their chosen number. Ask them to read the new number and to describe how the two numbers are alike and how they are different.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use large numbers from students' experiences, such as populations, video game scores, or salaries of professional sports figures and artists.
- Use visual models based on the cubic centimetre and cubic metre.
- Read and discuss children's books to explore number concepts.
- Provide students with frequent opportunities to read, write, and say numbers in standard and expanded form. (**Note:** Students should use proper spacing, not commas, when writing large numbers, and they should reserve the use of "and" for reading decimal numbers.)
- Discuss how large numbers can represent either a large amount or a small amount depending on the context used.
- Explore websites, such as Statistics Canada, to find examples of large numbers.

- Use various manipulatives (number cubes, spinners, number cards, etc.) to generate six-digit numbers. Students can then be asked to explore these numbers in many different ways.

SUGGESTED LEARNING TASKS

- Provide students with a place-value chart and counters. Ask them to model a six-digit number that has a 9 as two of its digits. Instruct students to add one more "counter" to a place that has a 9 and to write the new number with an explanation of how they found that number.
- Invite students to model a given five- or six-digit number with a place-value chart and counters. Ask them to explain why there are no counters in that place value on the place-value chart, but there is a digit in that place in the numeral. For example, if given the number 308 144, students would model it with a place-value chart and counters, and would explain why there are no counters in the ten thousand place on the place-value chart, but there is a digit in the ten thousand place in the numeral.
- Provide students with a set of counters and a place-value chart. Ask students to model three different five- or six-digit numbers. Then, ask students to write each of those numbers in expanded form.
- Give students a number in standard form (e.g., 34 360), and also in words. The word form should be incorrectly written (e.g., thirty thousand four hundred sixty). Ask students to correct the written words and to explain their thinking.
- Ask students to explain why the zeros are important in the number 23 006. Ask them to explain the affect on the value of the number if the zeros were removed?"
- Given a set of numbers, each containing a common digit in a different place, ask students to give the value of that digit in each number (e.g., 234 567, 108 300, 344 901).
- Given a set of five number cards 0–9, ask students to show the following:
  - the greatest possible number
  - the least possible number
  - a number with a value between the greatest and least
  - a number that is closer in value to the greatest than to the least
- Ask students to scan newspapers and magazines for large numbers and have them rewrite the headlines/sentences with the numbers written in words.
- Have students brainstorm and create headlines that include seven-digit numbers using words. Students may create computer-generated copies of their headlines.
- Ask students to locate large numbers in newspapers or magazines. Have them read, write, and represent those numbers in different ways.
- As a class, collect some type of object with the objective of reaching a specific quantity. For example, collect 25 000 buttons, pieces of junk mail, or beverage can tabs. If collecting objects is not possible, students could start a project in which they draw a specific number of dots each week until the objective is reached.
- Have students identify how many $100 bills it would take to make $1 000 000.
- Invite students to estimate how long a line of one million unit cubes would be.
- Ask students questions about the reasonableness of numbers such as, Have you lived 10 000, 100 000, or 1 000 000 hours yet? Are there 10 000, 100 000, or 1 000 000 people in any community in Nova Scotia? Have students explain their thinking.

- Create two-page spreads for a class book about one million. Each spread could begin, If you had a million _________, it would be__________. Alternatively the sentences could start, “I wish I had a million _________, but I would not want a million _________.
- Ask students to create six-digit numbers by rolling a number cube six times and then order the numbers from least to greatest.
- Invite students to explore the way numbers have been expressed in various types of media and personal conversations, and discuss why variations in saying and writing numbers might occur.
- Ask students to compare 10 000 steps to 10 000 metres. If you walked 10 000 steps per day, in how many days would you have walked 100 000 steps? One million steps?
- Ask students to list three non-consecutive numbers between 284 531 and 285 391.
- Invite students to place counters on a place-value chart to represent a number stated orally. The numeral can be written once the chart is filled in, and the number can be read back.
- Have students create a number line using a cash register tape. Ask them to place a four- or five-digit number on their number line and explain how they decided where to place their number on the number line.
- Tell students that Joanne decided that she wanted to raise money for the Terry Fox Foundation. Her goal was to raise $20 000 with the support of her school. The total funds raised were $17 692. A local company will round the donation to the nearest 1000 dollars. Ask students to tell how much the total donation will be.
- Invite students to work in pairs or in small groups. Provide each group with a sheet of chart paper. Ask each group to select a six- or seven-digit number and to represent that number in as many different ways as they can using expressions. After each group has completed as many different expressions for their chosen number as they can, post the chart paper from each group. Have the class examine each chart paper to determine the number represented by each of the expressions. Ask students to explain why all of the expressions on a sheet of chart paper are equal.
- Ask students to rename a number, less than 1 000 000, as the sum of other numbers.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- counters
- hundreds grids
- money
- number cards
- number line
- place-value charts

MATHEMATICAL LANGUAGE

Teacher	Student
▪ expanded notation	▪ expanded notation
▪ greater than, less than, equal	▪ greater than, less than, equal
▪ multiples	
▪ number expression	▪ number expression
▪ number lines	▪ number lines
▪ number words, symbols, digits	▪ number words, symbols, digits
▪ ones, tens, hundreds, thousands, million	▪ ones, tens, hundreds, thousands, million
▪ represent, partition numbers	▪ represent, partition numbers

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 137–143, 146
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 193–200, 201–202
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 19, 47–53
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 50, 51

Notes

SCO N02 Students will be expected to use estimation strategies, including front-end, front-end adjusted, rounding, and compatible numbers in problem-solving contexts. [C, CN, ME, PS, R V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N02.01 Provide a context for when estimation is used to make predictions, check the reasonableness of an answer, and determine approximate answers.
N02.02 Describe contexts in which overestimating is important.
N02.03 Determine the approximate solution to a given problem not requiring an exact answer.
N02.04 Estimate a sum, a difference, a product, or a quotient using an appropriate strategy.
N02.05 Select and explain an estimation strategy for a given problem.

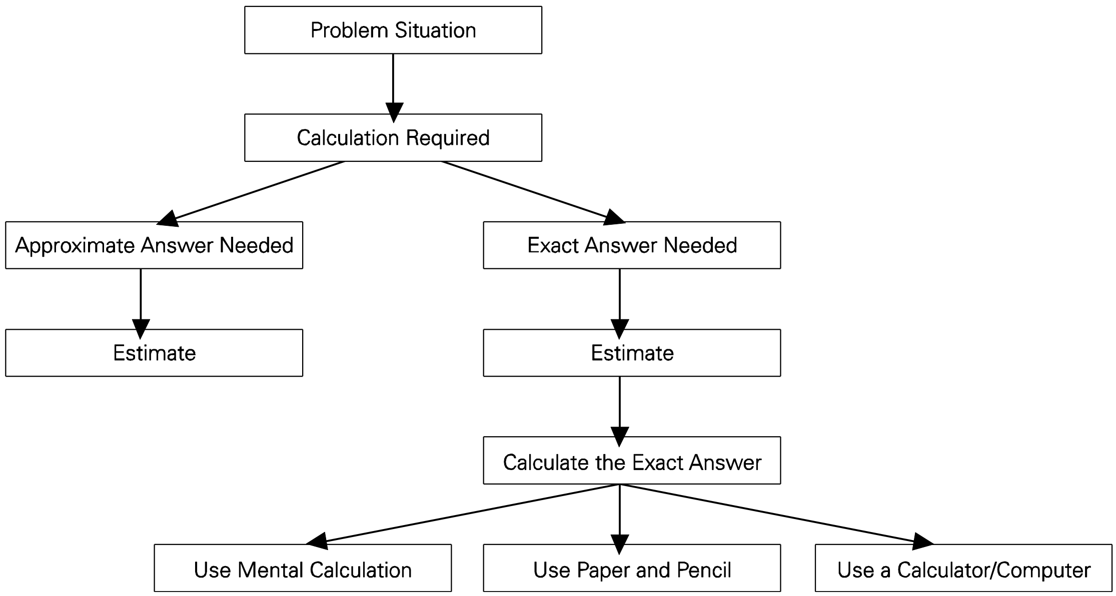
Scope and Sequence

Mathematics 4	**Mathematics 5**	**Mathematics 6**
N03 Students will be expected to demonstrate an understanding of addition and subtraction of numbers with answers to 10 000 (limited to three- and four-digit numerals) by ▪ using personal strategies for adding and subtracting ▪ estimating sums and differences ▪ solving problems involving addition and subtraction	**N02** Students will be expected to expected to use estimation strategies, including front-end, front-end adjusted, rounding, and compatible numbers in problem-solving contexts.	**N02** Students will be expected to solve problems involving whole numbers and decimal numbers.

Background

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated below.



Students need to recognize that estimation is a useful skill in their lives. However, to be efficient when mentally estimating sums, differences, products, and quotients, students must be able to access a strategy quickly, and they need a variety of strategies from which to choose. Students should be aware that in real-life estimation contexts **overestimating** is often important.

The context, the numbers, and the operations involved affect the estimation strategy chosen.

- **Rounding:** There are a number of things to consider when rounding. For example, when estimating a multiplication calculation, if one of the factors is a single digit, consider the other factor carefully. For example, when estimating 8 × 693, rounding 693 to 700 and multiplying by 8 is a much closer estimate than multiplying 10 by 700. Explore rounding one factor up and the other one down, even if it does not follow the "rounding rule" of going to the next closest multiple of 10 or 100. For example, when estimating 77 by 35, compare 80 × 30 and 80 × 40 to the actual answer of 2695.
- **Front-end estimation:** This strategy is the simplest of all the estimation strategies for addition, subtraction, and multiplication. It involves combining only the digits in the highest place value of each number to get an estimate. As such, these combinations will require only the use of the basic facts. While this strategy may be applied to division questions if the divisor is a factor of the highest place value of dividend, division estimation is better done by a rounding strategy.
  - sums (e.g., 253 + 615 is more than 200 + 600 = 800)
  - differences (e.g., 974 – 250 is close to 900 – 200 = 700)
  - products (e.g., the product of 23 × 24 is greater than 20 × 20 (400) and less than 25 × 25 (625))
  - quotients (e.g., the quotient of 831 ÷ 4 is greater than 800 ÷ 4 (200))
- **Adjusted Front-End Estimation:** This strategy is often used as an alternative to rounding to get closer estimates. It involves getting a Front-End estimate and then adjusting or compensating that estimate to get a better, or closer, estimate by either clustering all the values in the other place values to determine whether there would be enough together to account for an adjustment or considering the second highest place values. This second method of adjustment often results in a

closer estimate than first method and would likely only be bettered by the strategy of rounding to the two highest place values.

- **Compatible numbers:** Clustering compatible (or near compatible) numbers is useful for addition. When estimating the addition of a list of numbers, it is sometimes useful to look for two or three numbers that can be grouped to almost make 10s, 100s, or 1000s (compatible numbers). These pairs or trios of numbers provide estimates for 100 or 1000 and are combined with other estimates, as well as estimates of any leftovers, to get a total estimate for the list. For example, to solve 134 + 55 + 68 + 46, the 46 and 55 together make about 100; the 134 and 68 make about another 200 for a total of 300. Look for compatible numbers when rounding for a division estimate. For 477 ÷ 6, think "480 ÷ 6." For 332 ÷ 78, think "320 ÷ 80."

Please refer to Appendix A: Performance Indicator Background for a complete discussion of expectations for mental mathematics.

Students and teachers should note that multiplication and division estimations are typically further from the actual value because of the nature of the operations involved. Students should have opportunities to decide what might be the closest estimate, whether an estimate is over or under the exact answer, and to justify their thinking.

Students should continue to apply their understanding of addition and subtraction procedures for multi-digit calculations (limited to up to four-digit numbers) learned in earlier grades. In Mathematics 5, it is important for students to have opportunities to continue to apply and refine these strategies. Given that understanding large numbers and place-value concepts are focal ideas in Mathematics 5, students should be provided with opportunities to apply addition and subtraction estimation strategies while exploring large numbers.

Students should make use of place-value understandings in estimating or calculating and should be encouraged to talk about place-value concepts while explaining reasoning. For example, a student could be asked to estimate the difference between the average attendance at a major league baseball game (32 717) and an NHL hockey game (21 265). In estimating this difference, students might use front-end estimation and note that 30 thousand less 20 thousand is 10 thousand. Teachers should model the use of place-value language and encourage students to use place-value language when discussing their reasoning. Students should be encouraged to think about when paper and pencil or mental algorithms will work efficiently and when it may be more appropriate to use technology. The front-end example above may actually be done quicker mentally than with technology, and students should be encouraged to apply mental strategies when appropriate to develop their number and operation sense.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- You drink 250 mL of milk on the first day, 375 mL of milk the second day, and 450 mL of milk on the third day. About how many millilitres of milk did you drink during these three days? Stimulate the students' thinking by asking whether 900 mL would be a good estimate for the answer.
- Ask students to model the addition of 1273 and 2485 using concrete and/or visual representations and to record the process symbolically. Invite students to explain their method.
- Ask students to model the subtraction of 248 from 5073 using concrete and/or visual representations and to record the process symbolically. Invite students to explain their method.
- Ask students to create an addition or subtraction story problem for the number sentence 5330 – 185 = □ or 185 + □ = 2330.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students, Which pair of factors would you choose to estimate 37 × 94? Explain why.
  30 × 90   40 × 100   35 × 95   40 × 95   40 × 90
- Ask students to estimate each sum and explain their strategies.
  1976 + 3456   69 423 + 21 097
- Ask students to estimate each difference and explain their strategies.
  99 764 – 17 368   5703 – 755
- Ask students to add 6785 + 1834 and to explain how they know their answer is reasonable using estimates in their explanation.
- Ask students to solve problems that require an estimate such as, Jeff has 138 cans of soup. He wants to collect 500 cans for the food bank. About how many more does he need to collect?
- Tell students that a bus holds 58 passengers. Ask them to explain how they would estimate the number of people that could be transported in 18 buses.
- Tell students that you have multiplied a three-digit number by a one-digit number and the answer is about 1000. Ask the student to write three possible pairs of factors.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Support students in exploring strategies for estimation, then guide students toward more efficient and accurate strategies as needed. Ask, Is your estimation accurate? Why?
- Ask students to share estimation strategies. Begin with the least efficient strategies, then share progressively more complex strategies as this encourages participation from all and does not discourage others.
- Accept a range of estimates, and discuss why students got different estimates. Encourage students to focus on getting the "closest" estimate possible in a given context.
- Provide real-world contexts for estimations, as most situations require estimations and not precise answers.
- Practise strategy selection and explain the choice for estimation.

SUGGESTED LEARNING TASKS

- Tell students that □83 + 190 is about 600. Ask students to tell what digit should go in the box and to explain their thinking.
- Ask students to estimate what one might subtract in each case below so that the answer is close to, but not exactly, 50:
  - 384 – ____
  - 219 – ____
  - 68 – ____

- Invite students to describe a real-world situation where overestimating is appropriate.
- Ask students if they have lived closer to 400, 4000, or 40 000 days. Ask them to explain their thinking.
- Ask students to provide an estimate if a number between 300 and 400 is divided by a number between 2 and 7.
- Provide students with a variety of problems, such as the following:
  - On a trip you travel 4250 km the first week, 3755 km the second week, and 2115 km the third week. Estimate how many kilometres would be travelled during the three weeks. Explain whether the estimate is more or less than the calculated answer.
  - During one summer, Marcie travels 7185 km while Jimmy travels 4205 km. Estimate how much farther Marcie travelled than Jimmy during the summer. Explain the estimation strategy you used.
  - Tony has 375 baseball cards and 823 hockey cards. He estimates his total collection of sports cards to be 1100. Explain how Tony might have made his estimate closer to the actual total.
  - Judy used the following estimation strategy to estimate the sum of 365 and 437. Judy's thinking: I used the front-end rounding strategy. 365 is about 300 and 437 is about 400. 300 + 400 = 700. My estimate for the sum of 365 and 437 is about 700. Is Judy's estimate close to the actual sum? Is there an estimation strategy that Judy might have used to make her estimate closer to the calculated sum? Explain your thinking.
  - Juan used a calculator to find the product of 89 × 75. The calculator display shows the product as 66 750. Is the product shown on the calculator reasonable? Explain your thinking.
- Provide students with a variety of problems in which students are expected to choose from among three possible estimates. For example,
  - The table below shows how many people visited the Fall Fair in Antigonish in August 2010.

Day of the week	**Number of people**
Wednesday	124
Thursday	158
Friday	223
Saturday	378
Sunday	323

    Choose the best estimate for the total number of people that visited the Fall Fair on the weekend (Saturday and Sunday).
    700 1200 300
  - Sue ran 503 m. Luc ran 287 m. About how much farther did Sue run than Luc?
    800 m 300 m 100 m
  - A tree farmer sold 2352 Balsam fir trees for $8.50 a piece. About how much did he get for his trees?
    $2000 $20 000 $30 000
- Ask students to find two numbers with a difference of about 150 and a sum of about 500 or two numbers with a difference of about 80 and a sum of about 200.

Suggested Models and Manipulatives

- calculator
- number lines (including open number lines)

Mathematical Language

Teacher	Student
▪ approximate ▪ estimation ▪ front end, front-end adjusted, rounding, compatible numbers, compensation ▪ overestimating ▪ predictions ▪ reasonableness	▪ about ▪ estimation ▪ front end, front-end adjusted, rounding, compatible numbers, compensation ▪ overestimating ▪ predictions ▪ making sense

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 144–145, 160–161, 176–177
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 200–201, 216–217, 231–232
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 279–280

Notes

SCO N03 Students will be expected to describe and apply mental mathematics strategies and number properties to recall, with fluency, answers for basic multiplication facts to 81 and related division facts. [C, CN, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N03.01 Describe the mental mathematics strategy used to determine basic multiplication or division facts.
N03.02 Explain why multiplying by 0 produces a product of 0 (zero property of multiplication).
N03.03 Explain why division by 0 is not possible or is undefined (e.g., 8 ÷ 0).
N03.04 Quickly recall multiplication facts up to 9 × 9 and related division facts.

Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
N05 Students will be expected to describe and apply mental mathematics strategies, to recall basic multiplication facts to 9 × 9, and to determine related division facts.	**N03** Students will be expected to describe and apply mental mathematics strategies and number properties to recall, with fluency, answers for basic multiplication facts to 81 and related division facts.	**N03** Students will be expected to demonstrate an understanding of factors and multiples by ▪ determining multiples and factors of numbers less than 100 ▪ identifying prime and composite numbers ▪ solving problems using multiples and factors

Background

Outcome N03 is an extension of the Mathematics 4 outcomes, N04 and N05. In Mathematics 4, students were expected to recall the multiplication facts (up to 9 × 9) quickly and accurately by the end of the school year. In order to achieve this goal, students were introduced to a series of strategies, each of which addresses a cluster of multiplication facts. Each strategy was introduced, reinforced, and assessed before being integrated with previously learned strategies. Students were expected to understand the logic and reasoning of each strategy. Basic multiplication facts should be revisited in order to ensure that students have quick recall of these facts. This review will also be useful in using the “think multiplication” strategy for division facts.

The goal for Mathematics 5 is automaticity for both multiplication and division basic facts, which means that students should be able to recall multiplication and division facts with little or no effort. Fact recall should be automatic as a result of thinking about the relationship between the facts and extensive use of strategies. Students need to understand and use the relationship between multiplication and division and should recognize that multiplication can be used to solve division situations. Providing students with contextual problems to solve is a critical part of this process.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to explain how knowing 6 × 5 helps one to figure out 12 × 5.
- Ask students to explain how they would determine the answer to a division fact, (e.g., 30 ÷ 5 = ?) by relating it to multiplication.
- Ask students to recall basic multiplication facts (9 × 9) with a three- to five-second response time.
- Ask students to model, with counters, how to solve 12 ÷ 3.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask, If you buy muffins in boxes of 6, how many muffins are in 7 boxes? How would the number of muffins change if you bought 9 boxes? If you needed 36 muffins for a party, how many boxes would you buy?
- Tell students that Yan solved the problem 48 ÷ 8 by thinking, 48 ÷ 2 = 24, then 24 ÷ 2 = 12, and finally 12 ÷ 2 = 6. Ask them to explain the strategy Yan used.
- Ask students to explain how they could use multiplication to find the perimeter of a square.
- Ask students to use manipulatives to explain why 7 × 0 = 0 and 0 × 9 = 0.
- Have students use manipulatives to explain why 6 ÷ 0 is not possible.
- Tell students that you have 8 boxes, each of which holds 6 markers, and one other box that has only 5 markers in it. Ask students to describe at least two ways one could find the total number of markers, and to explain which strategy they would prefer and why.
- Invite students to share strategies to determine answers to unknown division facts.
- Have students fill in all the facts they know in a multiplication table. Ask them to identify what strategies might be used to fill in the rest of the table.

- Ask students to build a multiplication array to represent 5 × 2. Ask them to identify the related division facts represented by the array.
- Ask students to recall basic division facts with a three- to five-second response time.

Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Use a problem-solving context to introduce strategies and practise facts.
- Introduce and practise strategies. When students are proficient at more than one strategy, ask them explain why one strategy may be better than another in a given situation.
- Ask students to start with facts that they know. Allow them to use counters, base-ten blocks, colour tiles, and arrays as they continue to develop strategies.
- Ensure students understand why strategies work. Fact strategies should not become “rules without reasons” (Van de Walle and Lovin 2006b, 90).
- Provide students with situations that involve division by zero. For example, if you have 8 counters, how many sets of 0 can be made or how many times can you subtract 0 from 8 to get to 0 (8 ÷ 0)? This can also be explored using the relationship between multiplication and division. To solve 8 ÷ 0, students could try to use multiplication, and discover that there is no answer for $0 \times \square = 8$.
- Play games to practise strategies that lead to fact recall.
- To activate prior knowledge and connect multiplication and division facts, invite students to give the related division facts from flash cards showing different multiplication facts.
- Avoid using drill until students have mastered a strategy. Unless students have mastered a strategy, drills are not effective.

SUGGESTED LEARNING TASKS

- Invite students to use counters to model 6 × 6 in an array. Ask them to add another row or column to demonstrate a related fact and to explain their thinking.
- Ask students, If Jennifer reads a chapter of a novel each day, how many chapters will she have read in eight weeks? Ask students to explain the strategy used to determine the solution.
- Ask students to agree or disagree with this statement, “There are more than two ways to figure out any multiplication fact.” Invite students to use a fact of their choice to explain their thinking.
- Ask students if they agree or disagree with the following statement, If you know your multiplication facts, you already know your division facts. Invite students to provide a rationale for their answer.
- Provide small groups of students with a square piece of paper. Ask them fold the paper in half and record how many sections they have. Invite them to fold it again and record how many sections. Have them continue until they see a pattern of doubling. Relate this to halving.
- Use sets of “loop cards” (I have ___, who has ____) where the answer for one card answers the question on another to form a loop of questions and answers. For example, one card could read, “I have 24. Who has 3 × 4?” Another card would read, “I have 12. Who has 4 × 5.”
- Tell students that Caitlinn is stitching together a quilt for her grade 5 class using 30 quilt squares the students have prepared with their own drawings depicting friendship. Invite students to use 30 coloured tiles to model Caitlinn’s quilt design to determine all possible layouts. Ask them to identify all the all multiplication and related division facts represented by their designs.
- Ask students to use the repeated halving strategy to find the quotient of 48 ÷ 4.
- Tell students that Jamar solved the problem 32 ÷ 8 by thinking, 32 ÷ 2 = 16, then 16 ÷ 2 = 8, and finally 8 ÷ 2 = 4. Ask students to explain the strategy Jamar used.
- Ask students to explain how knowing that 56 ÷ 8 = 7 can help someone determine the quotient of 64 ÷ 8; 72 ÷ 8; or 80 ÷8.
- Ask students to explain how 24 coloured tiles could be used to show that 24 ÷ 6 = 4? Ask them to identify other division sentences that could be shown using the 24 coloured tiles.

SUGGESTED MODELS AND MANIPULATIVES

- area models
- array models
- base-ten blocks
- colour tiles
- counters
- number lines

MATHEMATICAL LANGUAGE

Teacher	Student
▪ factors, product ▪ groups of, rows of, jumps of ▪ mental mathematics strategy ▪ multiplication, division facts ▪ relating division to multiplication ▪ repeated addition, equal groups, number of groups ▪ repeated doubling, using halving, skip counting, ten facts, five facts (clock facts)	▪ factors, product ▪ groups of, rows of, jumps of ▪ mental mathematics strategy ▪ multiplication, division facts ▪ relating division to multiplication ▪ repeated addition, equal groups, number of groups ▪ repeated doubling, using halving, skip counting, ten facts, five facts (clock facts)

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 129–131
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 184–186
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 12, 74, 88–93

Videos

- *An Introduction to Teaching Multiplication Number Facts* (15:18 min.) (ORIGO Education 2010)
- *Teaching the Use-Ten Strategy for Multiplication Number Facts* (10:21 min.) (ORIGO Education 2010)
- *Teaching the Doubling Strategy for Multiplication Number Facts* (11:06 min.) (ORIGO Education 2010)
- *Teaching the Build-Up/Build-Down Strategy for Multiplication Number Facts* (16:01 min.) (ORIGO Education 2010)

Notes

SCO N04 Students will be expected to apply mental mathematics strategies for multiplication, including ▪ multiplying by multiples of 10, 100, and 1000 ▪ halving and doubling ▪ using the distributive property [C, ME, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Determine the products when one factor is a multiple of 10, 100, or 1000.
N04.02 Apply halving and doubling when determining a given product (e.g., 32 × 5 is the same as 16 × 10).
N04.03 Apply the distributive property to determine a given product that involves multiplying factors that are close to multiples of 10 (e.g., 98 × 7 = (100 × 7) – (2 × 7)).

Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
N05 Students will be expected to describe and apply mental mathematics strategies, to recall basic multiplication facts to 9 × 9, and to determine related division facts.	**N04** Students will be expected to apply mental mathematics strategies for multiplication, including ▪ multiplying by multiples of 10, 100, and 1000 ▪ halving and doubling ▪ using the distributive property	**N02** Students will be expected to solve problems involving whole numbers and decimal numbers.

Background

Students who can perform **mental computations** can determine answers without paper and pencil. This enhances flexible thinking. In Mathematics 5, students are extending the strategies learned in Mathematics 4 to multiply mentally. It is important to recognize that these strategies develop and improve over the years with regular practice. This means that mental mathematics must be a consistent part of instruction in computation in all grades. Mental strategies must be taught both explicitly as well as being embedded in problem-solving situations. Sharing of computational strategies within the context of problem-solving situations is essential.

Students should perform and discuss the following types of mental multiplication on a regular basis:
- multiplying by multiples of 10, 100, and 1000
- halving and doubling
- distributive property

Whenever presented with a problem that requires computation, students should be encouraged to first check to see if it can be done mentally. Students should select an efficient strategy that makes sense to them and consistently produces accurate results. Having students share the strategies that work best for them is essential to building a wide repertoire of strategies. A discussion could occur around which strategy, or combination of strategies, students find most useful.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to explain how knowing 8 × 10 helps someone find the product of 8 × 9.
- Ask students to recall the basic multiplication facts involving 9.
- Ask students to recall the basic multiplication facts involving 2.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Tell students that when asked to multiply 36 × 11, Kelly said, "I think 360 + 36 = 396." Ask students to explain Kelly's thinking.
- Ask students to explain why is it easy to mentally calculate the questions below.
  48 × 20 50 × 86
- Ask students to explain why Scott multiplied 11 × 30 to find the product of 22 × 15.
- Provide students with a problem situations to solve, such as the following:
  - Fourteen students raised $20 each in pledges for "Save the Wetlands Walk." How much money was raised? How much money would be raised if the pledges were increased to $50 each?
  - A hotel has 7 floors with 19 windows on each floor. How many windows are in the hotel? Explain how you know.

- Explain how you know that 48 × 50 is the same as 24 × 100.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with many experiences to construct a personal strategy and then be guided to use the most efficient strategy available. Mental mathematics strategies encourage students to think about the whole number and not just the digits.
- Provide students with frequent opportunities to share their mental mathematics strategies.
- Provide problem-based situations that support the use of mental mathematics strategies.
- Use materials and pictorial representations to demonstrate mental mathematics strategies.
- Introduce a strategy with the use of materials, practise the strategy, and continue to introduce and practise new strategies. When students have two or more strategies, it is important to encourage them to choose the most efficient strategy.
- Encourage students to visualize the process for the strategy they are using.
- Place students in pairs to practise strategies as well as strategy selection.
- Avoid timed tests until students have developed and practised specific mental mathematics strategies in other contexts.
- Ask students to keep track of when they use their mental mathematics strategies outside of the classroom and to write about these experiences.
- Ask students to keep a list of mental mathematics strategies that they regularly use.

SUGGESTED LEARNING TASKS

- Ask students to write a series of mental mathematics questions on recipe cards. Invite students to take the cards home to have a "race" with a parent/guardian. The student can then "teach" the strategy being practised at home.
- Ask students to explain how they could calculate 23 × 8 if the "8" key on the calculator was broken.
- Prepare cards with number sentences that can be solved using two or more mental mathematics strategies. Put these into a single package. Prepare simple pictures or labels for the strategies in the package. Ask students to sort the problems and then solve them using an appropriate strategy.
- Ask students to use square tiles to show that if the length of a rectangle is halved and the width is doubled, the area remains the same.
- Ask students to provide an explanation and examples for how to mentally multiply 99 by a one-digit number.
- Ask students to identify for which of the following computations halving and doubling would be an effective strategy to use. Ask them to explain their thinking.
  - 9 × 7
  - 8 × 13
  - 50 × 8
  - 51 × 9
  - 25 × 16
  - 35 × 4
- Give students the following equation: 57 × 7 = (60 × 7) ___ (3 × 7). Ask students to explain which operation should go in the ___ (+ or –) and to explain their thinking.
- Ask students to solve 68 × 7 using the distributive property and ask them to explain their method.

SUGGESTED MODELS AND MANIPULATIVES

- area model
- array model
- base-ten blocks
- counters
- place-value charts

MATHEMATICAL LANGUAGE

Teacher	Student
▪ distributive property ▪ mental mathematics strategy ▪ multiples of 10, 100, 1000 ▪ multiplication facts ▪ personal strategy ▪ repeated doubling, using halving	▪ distributive property ▪ mental mathematics strategy ▪ multiples of 10, 100, 1000 ▪ multiplication facts ▪ personal strategy ▪ repeated doubling, using halving

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 126, 129–131, 173–175
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 181, 184–186, 229–231
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 113, 116

Videos

- *An Introduction to Teaching Multiplication Number Facts* (15:18 min.) (ORIGO Education 2010)
- *Teaching the Use-Ten Strategy for Multiplication Number Facts* (10:21 min.) (ORIGO Education 2010)
- *Teaching the Doubling Strategy for Multiplication Number Facts* (11:06 min.) (ORIGO Education 2010)
- *Teaching the Build-Up/Build-Down Strategy for Multiplication Number Facts* (16:01 min.) (ORIGO Education 2010)

Notes

SCO N05 Students will be expected to demonstrate, with and without concrete materials, an understanding of multiplication (two-digit by two-digit) to solve problems. [C, CN, PS, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N05.01 Model the multiplication of two two-digit factors, using concrete and visual representations of the area model, and record the process symbolically.

N05.02 Illustrate partial products in expanded notation for both factors (e.g., For 36 × 42, determine the partial products for (30 + 6) × (40 + 2).).

N05.03 Represent both two-digit factors in expanded notation to illustrate the distributive property (e.g., To determine the partial products of 36 × 42, record
(30 + 6) × (40 + 2) = (30 × 40) + (30 × 2) + (6 × 40) + (6 × 2) = 1200 + 60 + 240 + 12 = 1512.).

N05.04 Describe a solution procedure for determining the product of two given two-digit factors, using a pictorial representation such as an area model.

N05.05 Solve a given multiplication problem in context, using personal strategies, and record the process.

N05.06 Create and solve multiplication story problems, and record the process symbolically.

N05.07 Determine the product of two given numbers using a personal strategy and record the process symbolically.

Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
N06 Students will be expected to demonstrate an understanding of multiplication (one; two-, or three-digit by one-digit) to solve problems by ▪ using personal strategies for multiplication, with and without concrete materials ▪ using arrays to represent multiplication ▪ connecting concrete representations to symbolic representations ▪ estimating products ▪ applying the distributive property	**N05** Students will be expected to demonstrate, with and without concrete materials, an understanding of multiplication (two-digit by two-digit) to solve problems.	**N02** Students will be expected to solve problems involving whole numbers and decimal numbers. **N08** Students will be expected to demonstrate an understanding of multiplication and division of decimals (one-digit whole number multipliers and one-digit natural number divisors).

Background

As for all computational questions, students should estimate before calculating. Immediate recall of basic multiplication facts is a necessary prerequisite not only for paper-and-pencil algorithmic procedures, but also for estimation and mental computation. Students should estimate products prior to exploring their methods or procedures for finding the product. Please refer to outcome N02 for expected estimation strategies.

In Mathematics 4, students were expected to symbolically multiply one-, two- and three-digit numbers by a one-digit number using reliable, accurate, and efficient personal strategies. In Mathematics 5, students will learn to multiply two two-digit numbers. Students should continue to use a variety of concrete and pictorial models to investigate multiplication to help them develop an understanding of the connection between the models and the symbols. Base-ten blocks serve as a tool for understanding the multiplication operation, and it is important that students use language as they manipulate the materials and pictorially record their work with base-ten blocks. It is important to start with a word problem and then have students use materials to model the problem and to determine the product. For example, There are 14 rows of chairs in the gym and there are 23 chairs in each row. How many chairs are in the gym? Students could use base-ten blocks or grid paper to model the situation.

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions, preferably choosing topics of interest to them. These opportunities provide students with a chance to practise their computational skills and clarify their mathematical thinking. To understand multiplication, students must have meaningful experiences with the many situations in which this operation is used. For addition and subtraction, students were exposed to 11 different structures of story problems. For multiplication (and division) there are three categories of story problem structures: equal groups, comparing, and combining.

The “equal groups” structure may be modelled with sets, arrays, area models, and linear or measurement models such as number lines. Students may use a set model when they need to find how many apples they will need to make eight bags of apples with a dozen apples in each bag for the school fundraiser (multiplication). A student might use an array model to determine how many chairs would be needed if 11 rows of 12 chairs were being set up for the concert. Students might use a number line when they need to know the distance they will travel if they make 12 jumps of three metres. Students should develop flexibility with modelling multiplication using these various representations.

Additionally, multiplication is used in “comparison” situations. Multiplicative comparisons lay the groundwork for proportional reasoning. With comparison, we may have a situation where we want to determine the size of a result, given the initial amount and the multiplier. Models used for comparison may involve sets, arrays, area models, and linear models such as number lines. For example, if a student is asked to build a tower that is eight blocks tall and then a tower that is four times as tall as the first tower, it would be appropriate to use a linear model such as linking cubes to show both towers. Students might also model this on a number line showing a distance of eight units repeated four times.

The third structure is the “combinations” structure, which provides the foundation for later work in probability. Combinations have only two sub-structures: finding the product given the size of the two sets, or finding the size of one set given the product and the other set. Commonly used models for combinations are tables. For example, if we know that Mike has three choices of lunch and four choices for a beverage, we can determine the number of possible lunches he can have using a tree diagram or a table.

	Milk	Orange juice	Water	Apple juice
Sandwich				
Salad				
Soup				

These structures of story problems have been given special attention here because these situational contexts help students to see the circumstances under which they might use multiplication and division. Students should be encouraged to both solve and create problems related to these structures. Students should solve problems by building and sketching models and explaining their discoveries both symbolically and verbally (either written or oral). Students should be exposed to multiplication and division situations that enable them to understand the various ways in which we use multiplication and division.

It is expected that, by the end of the year, students will be able to symbolically multiply two two-digit numbers using reliable, accurate, and efficient strategies. While some of these strategies may have emerged directly from students' work with base-ten blocks, other strategies should be modelled by students using the base-ten blocks to help understand the logic behind them. Students should be able to explain the strategy used and whether the solution is reasonable based on the prior estimate. Through the sharing of strategies, students will be exposed to a variety of possible multiplication strategies, and each student will adopt ones that he or she understands well and has made his or her own. That is why these strategies are often referred to as "personal strategies." The most appropriate strategy used may vary depending on the student and the numbers involved in the problem.

It is not expected that students would be explicitly taught all possible multiplication algorithms. Instead, teachers should provide opportunities for students to develop their personal algorithms for multiplication. Personal strategies make sense to students and are as valid as the traditional algorithm. Therefore, emphasis should be on students' algorithms rather than on the traditional algorithm. The paper-and-pencil recording of students' personal strategies should reflect their thinking and must be reliable, accurate, and efficient. Most important is that students can justify how and why an algorithm works. Students should be encouraged to refine their strategies to increase their efficiency, and teachers should monitor each student's symbolic recording of the strategy to ensure that the recording is accurate, mathematically correct, organized, and efficient. Students can begin by using concrete materials to solve the problem while the teacher assists by recording the student's thinking symbolically and facilitating the documentation. Through these opportunities to model and record solutions to story problems, students discover the most efficient algorithms for the numbers included in a given problem.

Strategies for multiplication can be more complex than those for addition and subtraction. Students need to be flexible in the way they think about the factors and should be thinking about numbers, not just digits. Students should have many opportunities to share their ideas and practise strategies.

There are many good reasons for students to be exposed to various algorithms for multiplication and for them to develop their own personal strategies and algorithms, including the following:

- One algorithm may be more meaningful to a student than another.
- One algorithm may work better for a particular set of numbers.
- Some algorithms lend themselves to mental computations.
- At home, parents may use a different algorithm than one taught at school, so students should be open to many strategies.

Students should be able to explain any algorithm they choose using correct mathematical language.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to model 24 × 6 with base-ten blocks. Ask them to explain their model.
- Ask students to model 3 × 125 with base-ten blocks. Ask them to explain their model.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to use a model to show how to find the total money collected for photos if 43 students each bring in $23.
- Ask students to explain why the product of two different two-digit numbers is always greater than 100.
- Have students draw an array to show 32 × 16, use the array to determine the product, and record the steps symbolically.
- Tell students that hardcover books were being sold at a book sale for $26. If 48 hardcover books were bought, how much money was spent?
- Ask students how far a cheetah can run in one minute if it runs 29 m per second. Have students explain their strategy for solving the problem.

- Prepare a series of two-digit by two-digit products and have students fill in missing numbers and provide justification for their choices. For example:

  74 × 32 = (70 + 4) × (___ + 2)
  = (70 × 30) + (___ × 2) + (4 × 30) + (4 × ___)
  = 2100 + 140 + ____ + ____
  = _____

- Show students the following:

$$\begin{array}{r} 41 \\ \times\ 24 \\ \hline 164 \\ 82 \\ \hline 246 \end{array}$$

  Ask students to explain the error and how to fix it.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Model multiplication concretely (base-ten blocks, grid paper).
- Use place-value language (e.g., 24 × 62 is twenty × 62 + four × 62).

- Use the language of multiplication, such as **factor**, **product**, and **distributive** and **commutative properties**. Effective communication of mathematical thinking should be done using words, pictures, and numbers. These should be logically outlined and clearly presented in students' responses.
- Ask students to estimate the product first in order to judge the reasonableness of the calculated product.
- Develop the symbolic representation from a model.
- Encourage frequent use of mental mathematics strategies.
- Guide students to use efficient strategies to perform calculations.

SUGGESTED LEARNING TASKS

- Provide students with a large rectangle (e.g., 24 cm × 13 cm). Have students fill the rectangle with base-ten materials to find the area. Have them write the related multiplication equation.
- Use known facts and combinations of facts that students know and apply them to more complex computations. For example, provide students with 31 × 24 and use 31 × 20, 31 × 4 and 30 × 24, 1 × 24 or other strategies to solve. Discuss which approach they preferred and why.
- Ask students to explore the pattern in these products: 15 × 15, 25 × 25, 35 × 35, etc. Have them describe the pattern and tell how the pattern could be used to predict 85 × 85 or 135 × 135. They might then test their predictions using a calculator. Alternatively, students might explore the pattern in these products: 19 × 21, 29 × 31, 39 × 41, and use it to make a prediction for 79 × 81 and 109 × 111.
- Find the product of 25 × 25. How can the product of 25 × 25 be used to help find the products of 25 × 24, 25 × 50, and 25 × 75?
- Ask students to solve problems that involve two-digit × two-digit multiplication and are relevant to their context. For example, all 27 students in the class each brought in $18 to help pay for a field trip. How much money should the teacher have collected if everyone brought in their money? Students should be given opportunity to create and solve their own and other students' problems.
- Discuss multiplication strategies. Have students share which strategies they prefer for particular situations and why.
- Ask students to explore the following: 24 + 35 is the same as 25 + 34. Is 24 × 35 the same as 25 × 34? Invite students to provide an explanation.
- Have students explain at least two different strategies they would use to solve a given problem. Ask the student which of the strategies they would prefer and why. (Record observation about the efficiency of the strategy chosen. This would be an opportunity to discuss the appropriate use of a given strategy with the student.
- Using ten numeral cards 0–9 invite students to select four cards to create two two-digit numbers. These numbers will be used as factors in a multiplication problem. Ask students to use the two two-digit numbers to create and solve a word problem that can be solved using multiplication.
- Ask students to model 31 × 24 using base-ten blocks. Ask them to represent the model on grid paper, clearly indicating the partial products and the final product. Finally, ask them to solve the problem symbolically. Ask students to explain how the three representations of the question are the same.
- Present students with the following problem: The fish processing plant finished packaging 25 crates of halibut. There were 72 kg of halibut in each crate. How many kilograms of halibut were packaged altogether? Use words, numbers, and pictures to solve the problem.

- Ask students to calculate the products of 25 × 36, 14 × 23, 22 × 32, and 11 × 17 and to record their process using numbers, words, and/or pictures.
- On chart paper prepare a series of two-digit by two-digit products and ask students to fill-in missing numbers and provide justification for their choices. For example,

  45 × 36

  = (40 + 5) × (__ + 6)

  = (40 × 30) + (40 × __) + (5 × __) + (5 × 6)

  = 1200 + ___ + 150 + 30 = ____
- Ask students to use a model to show the amount of money collected for photographs if 43 students each bring in $23.
- Tell students that Noah planted 15 rows of tulips with 24 tulips in each row. When determining how many tulips he planted, Noah wrote the following:

$$\begin{array}{r} 15 \\ \underline{\times\ 24} \\ 20 \\ 40 \\ 10 \\ \underline{+\ 200} \\ 270 \end{array}$$

  Ask students to explain whether Noah correctly calculated the number of tulips he planted. If Noah was incorrect, ask students to correct his error.

Suggested Models and Manipulatives

- base-ten blocks
- calculators
- grid paper

Mathematical Language

Teacher	Student
▪ area model, sets, arrays, number lines ▪ distributive property ▪ estimate ▪ expanded notation ▪ factors, product ▪ partial product	▪ area model, sets, arrays, number lines ▪ distributive property ▪ estimate ▪ expanded notation ▪ factors, product ▪ partial product

Resources/Notes

Print

- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 116–117, 120, 129–130
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 52–53, 55

Videos

- *Using Mental Strategies to Multiply* (26:16 min.) (ORIGO Education 2010)
- *Using Language Stages to Develop Multiplication Concepts* (16:97 min.) (ORIGO Education 2010)

Notes

SCO N06 Students will be expected to demonstrate, with and without concrete materials, an understanding of division (three-digit by one-digit), and interpret remainders to solve problems. [C, CN, PS]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N06.01 Model the division of two given numbers, using concrete or visual representations, and record the process symbolically.

N06.02 Explain that the interpretation of a remainder depends on the context.
- Ignore the remainder (e.g., making teams of four from 22 people [five teams, but two people are left over]).
- Round the quotient up (e.g., the number of five-passenger cars required to transport 13 people).
- Express remainders as fractions (e.g., five apples shared by two people).
- Express remainders as decimals (e.g., measurement and money).

N06.03 Solve a given division problem in context, using personal strategies, and record the process.

N06.04 Create and solve division story problems, and record the process symbolically.

N06.05 Determine the quotient of two given numbers using a personal strategy and record the process symbolically.

Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
N07 Students will be expected to demonstrate an understanding of division (one-digit divisor and up to two-digit dividend) to solve problems by ▪ using personal strategies for dividing, with and without concrete materials ▪ estimating quotients ▪ relating division to multiplication	**N06** Students will be expected to demonstrate, with and without concrete materials, an understanding of division (three-digit by one-digit), and interpret remainders to solve problems.	**N02** Students will be expected to solve problems involving whole numbers and decimal numbers. **N08** Students will be expected to demonstrate an understanding of multiplication and division of decimals (one-digit whole number multipliers and one-digit natural number divisors).

Background

Students should estimate quotients prior to exploring their methods or procedures for calculating the quotient. Please refer to outcome N02 for estimation strategies.

In Mathematics 4, students developed an understanding of the meaning of division. They drew on their conceptual knowledge to use models and pictures in developing strategies to solve problems involving two-digit dividends and one-digit divisors. They learned that division can be modelled with sets, arrays, area models, and number lines. In Mathematics 5, they will extend this understanding to solve problems involving up to a three-digit dividend and a one-digit divisor, and they will continue to use models in determining quotients.

The concept of division needs to be taught in conjunction with multiplication. The teacher should ensure that students recognize that multiplication and division are two ways of looking at the same situation—this is very clear when they examine models or pictures. Some students might think, What do I multiply 3 by to get 18? when asked to find 18 ÷ 3. Other students might imagine the area model and think, How many will be in each row if I organize 18 objects into 3 rows? Allowing students multiple opportunities to make connections between multiplication and division, and the concrete and pictorial representations of these operations, will help students to develop understanding of the operations.

Students should use a variety of models to investigate division and develop an understanding of the connection between the models and the symbols. Base-ten blocks serve as a tool for understanding the division operation, and it is important that the students use language as they manipulate the materials and record their work with base-ten blocks pictorially. Students may be introduced to division by asking them to solve a problem such as the following: There are 287 students in the school. The principal makes teams of 9 for a school fun day. How many teams can the principal make? Allowing the students to solve the problem using base-ten blocks provides opportunities for personal strategies to be developed.

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions, preferably choosing topics of interest to them. These opportunities provide students with a chance to practise their computational skills and clarify their mathematical thinking. To understand division, students must have meaningful experiences with the many situations in which this operation is used. For division (and multiplication) there are three categories of story problem structures: equal groups, comparing, and combining. These structures of story problems should be given special attention because these situational contexts help students to see the circumstances under which they might use multiplication and division. Students should be encouraged to both solve and create problems related to these structures. Students should solve problems by building and sketching models and explaining their discoveries both symbolically and verbally (either written or oral). Students should not be expected to do symbolic manipulation in isolation. Students should be exposed to multiplication and division situations that enable them to understand the various ways in which we use multiplication and division. Multiplication and division situations should involve sets, arrays, area models, and linear models.

It is not expected that students would be explicitly taught all possible division algorithms. Instead, teachers should provide opportunities for students to develop their personal algorithms for division. Students can begin by using concrete materials to solve the problem while the teacher assists by recording the thinking symbolically and facilitating the documentation. Through these opportunities to model and record solutions to story problems, students discover most efficient algorithms for the numbers included in a given problem. Two possible strategies and symbolic recordings are described on the following page. Additional algorithms are described in the appendix.

Sharing model with written record

453 ÷ 3 =
Make 100 sets of 3, using 300; 153 left. Make 50 sets of 3 using 150; 3 left …

The number of sets at each stage tends to be a multiple of 10 or 100 to facilitate computation.

151	
3)453	
− 300	100
153	50
− 150	1
3	

Distributive property (partition numbers)

453 ÷ 3 =
Think, 453 = 300 + 150 + 3
(300 ÷ 3) + (150 ÷ 3) + (3 ÷ 3)
100 + 50 + 1 = 151

When dividing whole numbers, there are often remainders. Students must understand what these remainders mean, as well as how to express them symbolically. The context of **remainders** must be discussed with students. They must understand why the number of units leftover after the sharing must be less than the **divisor**. Models help to clarify this idea. Students need many opportunities to explore the different interpretations of the remainder in problem solving situations to decide if it should be **ignored**, **rounded up**, expressed as a **fraction** or a **decimal**. A common mistake of students is to write a remainder as a decimal even when the divisor is not 10, (e.g., a remainder of 7 is written as 0.7). This should be addressed through a discussion of remainders and the meaning of tenths.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to use models or draw pictures to show 83 ÷ 3 and explain their thinking.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to use base-ten blocks to model how to divide 489 by 7.
- Tell students that Dmitry solved the following problem: There were 115 people going to a soccer game. Each mini-bus can carry 8 people. How many mini-buses are needed? Dmitry's final answer was 15. Ask students to explain how Dmitry may have solved the problem.
- Ask students to create and solve a problem involving division with a divisor of 6 and a dividend of 252.
- Tell students that at the "T-Shirt Shop," you can buy T-shirts in packages of 8. One package costs $130. At "Big Deals," a T-shirt costs $18. Does "Big Deals" have the better price? How do you know? Invite students to record and explain their process.
- Tell students that Mackenzie solved a word problem by dividing 288 by 4. She said the answer was 72. Ask students to explain what the problem might have been.
- Tell students that Lee is a farmer. He has 324 metres of fencing material to build a new space for his animals. He wants each side of the space to be the same length. What are three different possible spaces that he could make? How many sides would there be in each one and how much fencing would be left over?
- Ask students to identify in which of the following situations you would ignore the remainder, round up the quotient, or express as a fraction.
  - William has 185 hockey cards that he wants to share equally among his three friends. How many cards will each person receive?
  - Mrs. Cormier has 9 granola bars to share equally among her 4 nephews. How many granola bars will each nephew receive?
  - Yuma can transport 3 people in his canoe. How many trips would it take him to transport 35 people across a river?

  Ask students to explain their thinking.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

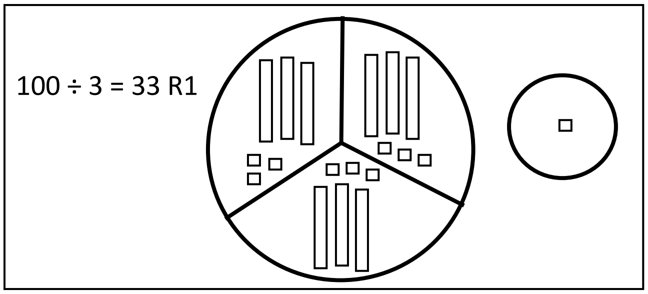
Consider the following strategies when planning daily lessons.

- Provide students with the opportunity to solve division problems using base-ten materials and other materials.
- Provide students with the opportunity to partition numbers to solve problems (e.g., For 92 ÷ 4, think 92 = 80 + 12, so 80 ÷ 4 = 20 and 12 ÷ 4 = 3, so the quotient is 23).
- Present division questions in problem-solving situations.
- Provide regular practice and discussion of estimation strategies to support division.
- Ask students to create, share, and solve problems involving division.
- Use multiplication to help estimate and solve division questions. For example to solve 448 ÷ 7, think, how many groups of 7 would be close to 448? Sixty groups would be 420 and 70 groups would be 490 which is more than 448. The quotient must be between 60 and 70. Since 448 is 28 more than 420, it would be possible to make 4 more groups and the quotient would be 64.

SUGGESTED LEARNING TASKS

- Ask students to write a word problem involving division where their interpretation of the remainder would be
  - a situation in which the remainder would be ignored
  - a situation in which the remainder would be rounded up
  - a situation in which the remainder would be part of the answer
- Tell students that a scientist discovered a group of creatures in the Bay of Fundy. The total number of legs was 84. If each creature had the same number of legs, how many creatures were there and how many legs were on each creature? Ask students to identify at least two different possibilities and explain using words and pictures.
- Present students with problems such as those below and ask them to use base-ten blocks, pictures, and numbers to solve the problems.
  - There are 253 children going on a field trip. If each van can carry 6 children, how many vans are needed?
  - The teacher asked some students to put 389 empty juice bottles into cartons to be recycled. If each carton holds nine bottles, how many cartons will be filled?

- Ask students to tell what division is being modelled below and to provide a word problem that would apply to the model.



- Ask students to use base-ten blocks to model 253 shared equally among 7 groups and to represent their solution using diagrams and a number sentence.
- Tell students that Jazmin solved the following problem: There were 367 fans going to a hockey game. Each SUV can carry 7 fans. How many SUV are needed? Jazmin's determined that 367 ÷ 7 = 52 R3. Ask students to explain what the remainder 3 represents. Tell students that Jazmin's answer to the question was, 53 SUVs are needed to carry the fans. Ask students to explain the reason for Jazmin's answer.
- Ask students to use their knowledge of basic facts and place value to help them calculate each of the following:

  240 ÷ 8 = ___

  560 ÷ 7 = ___

  480 ÷ 6 = ___

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- counters
- linking cubes
- money

MATHEMATICAL LANGUAGE

Teacher	Student
▪ area model ▪ estimating ▪ interpret remainders ▪ quotient, divisor, dividend ▪ relating division to multiplication	▪ area model ▪ estimating ▪ interpret remainders ▪ quotient, divisor, dividend ▪ relating division to multiplication

Resources/Notes

Print

- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 121–127
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 59–61

Notes

SCO N07 Students will be expected to demonstrate an understanding of fractions by using concrete, pictorial, and symbolic representations to ▪ create sets of equivalent fractions ▪ compare and order fractions with like and unlike denominators [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N07.01 Represent a given fraction of one whole, set, linear model, or region using concrete materials.
N07.02 Create a set of equivalent fractions, and explain, using concrete materials, why there are many equivalent fractions for any given fraction.
N07.03 Model and explain that equivalent fractions represent the same quantity.
N07.04 Determine if two given fractions are equivalent, using concrete materials or pictorial representations.
N07.05 Identify equivalent fractions for a given fraction.
N07.06 Compare and order two given fractions with unlike denominators by creating equivalent fractions.
N07.07 Position a given set of fractions with like and unlike denominators on a number line, and explain strategies used to determine the order.
N07.08 Formulate and verify a personal strategy for developing a set of equivalent fractions.

Scope and Sequence

Mathematics 4	**Mathematics 5**	**Mathematics 6**
N08 Students will be expected to demonstrate an understanding of fractions less than or equal to one by using concrete, pictorial, and symbolic representations to ▪ name and record fractions for the parts of one whole or a set ▪ compare and order fractions ▪ model and explain that for different wholes, two identical fractions may not represent the same quantity ▪ provide examples of where fractions are used	**N07** Students will be expected to demonstrate an understanding of fractions by using concrete, pictorial, and symbolic representations to ▪ create sets of equivalent fractions ▪ compare and order fractions with like and unlike denominators	**N04** Students will be expected to relate improper fractions to mixed numbers and mixed numbers to improper fractions.

Background

In Mathematics 4, students created fractions focusing on parts of one whole and parts of a set. In Mathematics 5, they will continue to work with fractions of one whole and of a set and will be expected to find equivalent fractions. Most of the work done by students will involve representing fractions concretely, pictorially, and symbolically.

Developing number sense with fractions takes time, and it must be supported with a conceptual approach and the use of materials. Students should have opportunities to use area, set, and linear models for fractions. Using a variety of manipulatives helps students understand properties of fractions and focus on the relationship between the two numbers in a fraction. It is important for students to understand that fractions do not indicate anything about the size of the whole they are describing. A common error made by students at this level is to think, because of their experience comparing whole numbers, that a larger denominator means the fraction is larger (e.g., they think $\frac{4}{7}$ is greater than $\frac{4}{6}$).

Students should continue to use conceptual methods to compare fractions. These methods include the following:

- Comparing each to a **benchmark** (e.g., $\frac{2}{5}$ is more than or less than $\frac{1}{2}$).
- Comparing the two **numerators** when the fractions have the same denominator.
- Comparing the two **denominators** when the fractions have the same numerator.

Considerable time needs to be spent on activities and discussions to develop a strong number sense for fractions. Provide students with a variety of experiences using different models (number lines, pattern blocks, counters, fraction strips, etc.) and different representations of one whole with the same model. Students should recognize that a fraction can **name part of a set** as well as **part of one whole** and the size of these can change. Students also need to understand that fractions can only be compared if they are parts of the same whole. One-half of a watermelon cannot be compared to one-half of an orange. When comparing one-half and one-fourth, the whole is the “unit” (1).

It is important that students are able to **visualize equivalent fractions** as the naming of the same **region** or **set** partitioned in different ways. Students should be given opportunities to explore and develop their own strategies for creating equivalent fractions. They should be able to explain their strategy to others. Rules for multiplying numerators and denominators to form equivalent fractions should not be provided to students to follow without a conceptual understanding of why they work. Students must use concrete models to develop their understanding of sets of equivalent fractions. When formulating and verifying a rule for developing a set of equivalent fractions, students should recognize the multiplicative nature between the numerator and denominator.

At this grade level, students will be working with fractions that are less than or equal to one. Students will not work with mixed numbers or improper fractions until Mathematics 6.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Place the following pairs of fractions before students, one at a time. Tell students to circle the larger fraction and to explain in words how they know that the fraction is larger. Then, have them select a manipulative and model the fractions to verify their selection.

  $\frac{1}{5}$ $\frac{3}{5}$ $\frac{3}{8}$ $\frac{3}{5}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{4}{8}$ $\frac{3}{6}$ $\frac{3}{4}$ $\frac{9}{10}$

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to create a diagram or use a model to show why $\frac{4}{8}$ and $\frac{1}{2}$ are equivalent.
- Invite students to explain the meaning of equivalent fractions using words, numbers, and pictures.
- Provide students with a set of equivalent fractions, such as $\frac{1}{3}, \frac{2}{6}, \frac{4}{12}, \frac{8}{24}, \frac{16}{48}$. Have students describe a pattern for the set of fractions.
- Ask students to use their fingers and hands to show that $\frac{1}{2}$ and $\frac{5}{10}$ are equivalent fractions. Alternatively, students might be asked to choose a different model or manipulative to show this or another equivalence.
- Have students place the following fractions on a number line: $\frac{1}{2}, \frac{9}{10}, \frac{4}{5}, \frac{1}{5}$. Ask them to explain the strategy they used to determine the location of each fraction.
- Invite students to make a diagram and to show that $\frac{10}{15} = \frac{2}{3}$.

- Ask students to write two equivalent fractions for the following diagram. Ask them to show their work pictorially and symbolically.

- Invite students to use various manipulatives to create as many different equivalent fractions for $\frac{2}{3}$ as they can.
- Ask students to explain, in writing, the meaning of equivalent fractions using words, numbers, and pictures.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

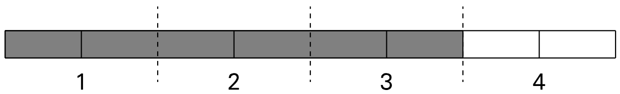
Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with a variety of activities that include the three interpretations of fractions:
  (1) part of a whole (e.g., serving of lasagna)
  (2) part of a set (e.g., part of 30 marbles)
  (3) part of a linear measurement (e.g., part of a 4 m piece of rope)
- Provide many opportunities for students to model fractions both concretely and pictorially, using a variety of models, such as pattern blocks, grid paper, fraction pieces, fraction towers, counters, Cuisenaire rods, egg cartons, and number lines.

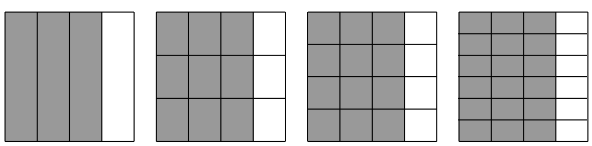
- Point out to students that to rename $\frac{6}{8}$ as $\frac{3}{4}$, you can "clump" the eight sections of the whole into twos. There are then four groups of two sections; three of the four groups are shaded.



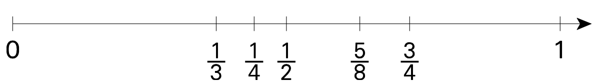
- Use number lines and other models to compare fractions and explore equivalencies.

SUGGESTED LEARNING TASKS

- Invite students to fold a piece of paper into fourths and to colour $\frac{1}{4}$. Ask students to fold the paper again. Have them explain what equivalent fraction is represented? Ask students to fold the paper again. Ask them to explain what equivalent fraction is now represented? Discuss the pattern.
- Ask students to prepare a poster showing equivalent fractions. They should determine equivalent fractions using pattern blocks.
- Give students a sheet with four squares. Ask them to shade $\frac{3}{4}$ on each square vertically. Then, ask them to subdivide each square with a different number of horizontal lines. Ask them to use the resulting pictures to find possible equivalent fractions for $\frac{3}{4}$.



- Provide students with a number line that has one of the fractions placed incorrectly. Ask students identify the error and provide an explanation for where it should be correctly placed.



- Tell students that John ate $\frac{4}{8}$ of a pizza and Joanne ate $\frac{1}{2}$ of a pizza. Ask them to explain which person ate more pizza.
- Ask students to explain, using words, pictures, and concrete materials, whether $\frac{2}{3}$ and $\frac{3}{5}$ are equivalent.

Suggested Models and Manipulatives

- colour tiles
- counters
- Cuisenaire rods
- dominoes
- double number lines
- egg cartons
- fraction circles
- fraction pieces
- fraction strips
- geo-boards
- grid paper
- number lines
- pattern blocks

Mathematical Language

Teacher	Student
▪ compare, order ▪ equivalent fractions ▪ number line ▪ numerators, unlike denominators ▪ same quantity ▪ whole, set, linear model, region	▪ compare, order ▪ equivalent fractions ▪ number line ▪ numerators, unlike denominators ▪ same quantity ▪ whole, set, linear model, region

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 200–202, 203–206, 207–209
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 254–256, 257–260, 262–263
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 146–149, 152
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 75–78

Notes

SCO N08 Students will be expected to describe and represent decimals (tenths, hundredths, and thousandths) concretely, pictorially, and symbolically. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

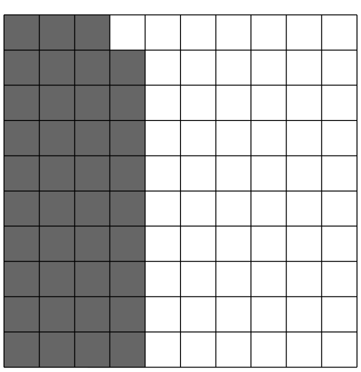
N08.01 Write the decimal for a given concrete or pictorial representation of part of a set, part of a region, or of a unit of measure.
N08.02 Represent a given decimal using concrete materials or a pictorial representation.
N08.03 Represent an equivalent tenth, hundredth, or thousandth for a given decimal, using concrete or visual representations.
N08.04 Express a given tenth as an equivalent hundredth and thousandth.
N08.05 Express a given hundredth as an equivalent thousandth.
N08.06 Explain the value of each digit in a given decimal.

Scope and Sequence

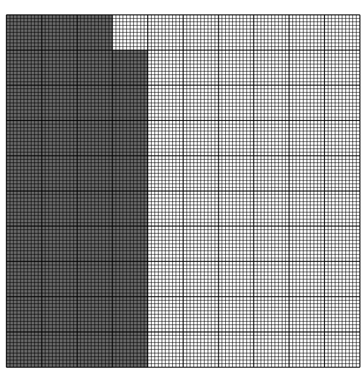
Mathematics 4	Mathematics 5	Mathematics 6
N09 Students will be expected to describe and represent decimals (tenths and hundredths), concretely, pictorially, and symbolically.	**N08** Students will be expected to describe and represent decimals (tenths, hundredths, and thousandths) concretely, pictorially, and symbolically.	**N01** Students will be expected to demonstrate an understanding of place value for numbers greater than one million and less than one thousandth.

Background

In Mathematics 4, students were introduced to decimal tenths and hundredths. In Mathematics 5, students continue to use physical materials to represent or model decimal tenths, hundredths, and thousandths. In this way, they can better see the relationship between **hundredths** and **thousandths**. For example, students might use a thousandth grid (the same size as a hundredth grid) to model decimals to the thousandths.

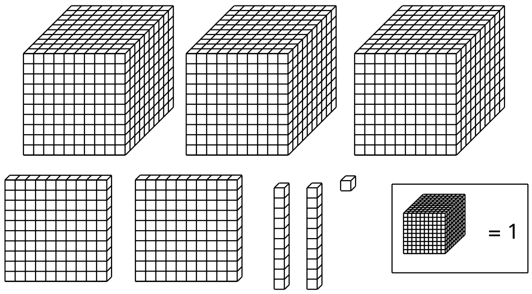


0.39 of the square



0.390 of the square

Alternatively, base-ten blocks might be used to illustrate the relationship. Within a given context, the large block could represent 1, and then the flat would represent 0.1, the rod 0.01, and the small cube 0.001. For example, 3.231 could be modelled as shown.



Varying which block represents one whole helps students develop flexibility in their thinking about decimal fractions. Students sometimes struggle with the concept that thousandths are smaller than tenths and hundredths based on their previous knowledge that thousands are larger than tens and hundreds. It is important that students recognize that decimals extend the place-value system to represent the parts of one whole. While money is commonly used as a representation for decimals, it typically only represents tenths and hundredths. Students do not usually think about the cents as being part of the whole. Students can represent thousandths using **length measurements**, since 1 mm = 0.001 m. For example, 0.423 m can be represented as 423 mm, 42.3 cm (a little more than 42 cm).

Like fractions, decimals have multiple names, and students must become proficient at representing and naming them in a variety of ways (e.g., 5.67 could be read as five and sixty-seven hundredths or fifty-six tenths, 7 hundredths). Provide opportunities for students to read decimals in context. Saying decimals correctly will help students make the connection between decimals and fractions (SCO N09). For example, 3.147 should be read as three **and** one hundred forty-seven thousandths, and not as three point one four seven.

Students should recognize that thousandths can represent something quite small or something very large depending upon the context in which they are used. For example, 0.025 m is only 2.5 cm, which is a small measurement; however, 0.025 of the population of Canada refers to 25 out of every one thousand people, and 25 000 out of every one million people, or a very large number of people. Discussions such as these help students to develop greater number sense.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

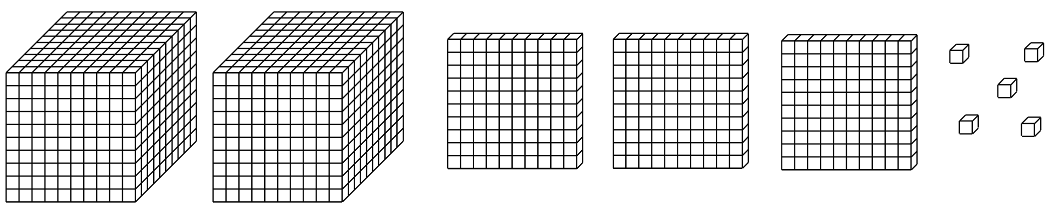
Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to use models of their choice to explain why 0.40 and 0.4 are equivalent.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to express 0.135 at least three different ways (e.g., one tenth, three hundredths, five thousandths; thirteen hundredths, five thousandths; one hundred thirty-five thousandths).
- Tell students that gasoline is priced at 83.9¢ per litre. Ask, What part of a dollar is this?
- Ask students to write 10 different decimal numbers that have tenths, hundredths, and/or thousandths. Have them draw pictures of base-ten blocks that would represent their numbers.
- Present students with a base-ten model of decimal numbers and ask the student to represent the model with a decimal number.



- Ask students to use hundredth and thousandth grids or base-ten blocks to model equivalent decimals such as 0.49 and 0.490.
- Show students cards on which decimal numbers have been written (e.g., 0.4 m, 0.75 m and 0.265 m). Ask students to place the cards on a metre stick at the correct location.
- Give students a thousandth grid and ask them to shade in given decimal numbers (e.g., 0.247).
- Ask students to write the numerals for "two hundred fifty-six thousandths" and "two hundred and fifty-six thousandths." Ask students to explain why watching and listening for "and" is important when interpreting numbers.
- Give students three number cubes. Ask them to make the greatest and least possible decimal numbers using the numbers rolled as the digits. Invite students to read the decimal numbers aloud.
- Ask students to describe the meaning of each digit in a given decimal (e.g., 6.083).
- Present students with the number 0.5 and ask them to read the number in at least two different ways. Ask them to explain their thinking.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Write decimals using place-value language and expanded notation to help explain equivalence of decimals.

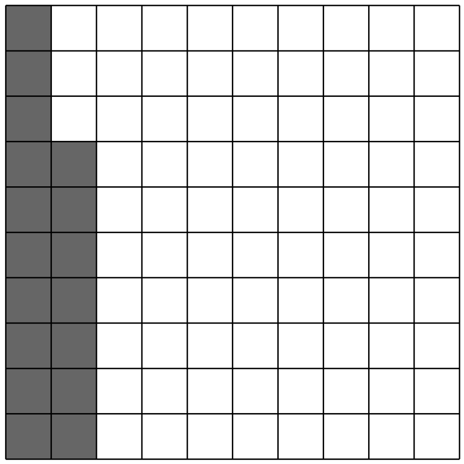
  0.4 = 4 tenths
  0.40 = 4 tenths + 0 hundredths
  0.400 = 4 tenths + 0 hundredths + 0 thousandths

  Since adding zeros has no effect, 0.4 must equal 0.40 and 0.400
- Use the same sized tenths, hundredths, and thousandths grid squares to draw equivalent decimals.
- Help students extend the place-value system to decimals by focusing on the basic pattern of ten. While building on their understanding of tenths and hundredths from Mathematics 4, students need to know that it takes 1000 equal parts (thousandths) to make one whole. Explore the pattern of the place-value names (whole numbers and decimals).
- Vary the representation of the whole. Use a cube, flat, and rod to represent the whole in different situations. Students often have a fixed notion of what these models represent, and it is important to reinforce the idea that a decimal relates a part to a whole the same way that fractions do.
- Provide a variety of models, stressing the magnitude of the number. For example, 0.452 could be modelled using a number line (about one half), base-ten blocks, a thousandth grid, or a place-value chart.

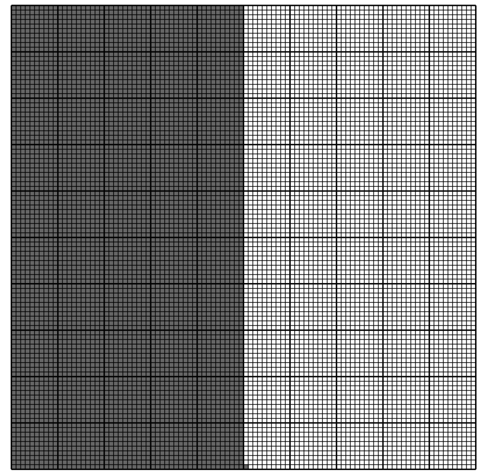
SUGGESTED LEARNING TASKS

- Present students with pictures, such as those shown below, and ask them to write a fraction and a decimal to show the shaded part of each of the diagrams:

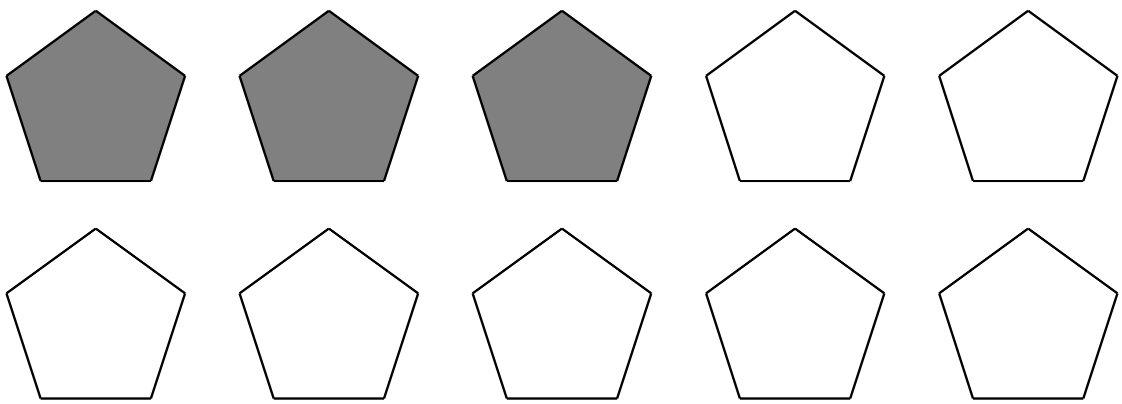
A.



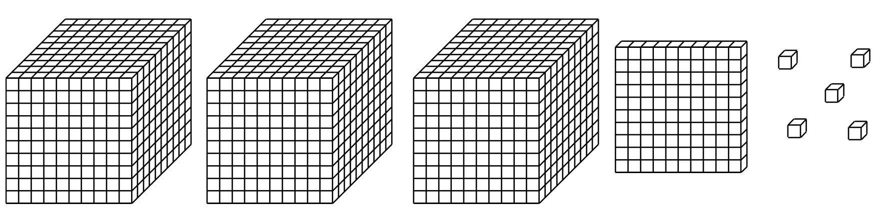
B.



C.



- Present a riddle to the class such as, I have 25 hundredths and 4 tenths. What am I? Invite students to use a model of their choice to represent the solution to the riddle.
- Present students with a base-ten model or picture of decimal numbers, and ask them to represent the model with a decimal number.



- Ask students to identify tenths, hundredths, and thousandths of a metre on a metre stick. Then, invite students to measure objects to the nearest tenth (decimetre), hundredth (centimetre), and thousandth (millimetre) of a metre.
- Invite students to model, using a thousandth grid, the numbers 0.3, 0.30, and 0.300 and to explain why they are equivalent.

- Make sets of cards showing decimals in different forms including expanded form, pictorial representations, and equivalent decimals. Invite students to play matching games or decimal snap.
- Provide opportunities for students to find and share how large numbers are represented in newspapers and magazines (e.g., an executive's salary may be written as 4.5 million dollars).
- Place five different displays of combinations of base-ten blocks. Ask students to visit each display and record the five decimal numbers.
- Provide students with two hundredths circles, each of a different colour. Cut each disk along one radius so they can be fit together. Students can use these to model given decimals, or to write decimals from a given model. Please see Van de Walle and Lovin, Volume 2, 2006, page182, for a full description of this activity.
- Use the calculator to "count." Enter 0.1 + 0.1 =, + 0.1 =, =, =, … When the display shows 0.9, have students predict what number will be next. Extend this to use 0.01 and 0.001 to demonstrate the relative magnitude of hundredths and thousandths.
- Ask students to identify a situation in which 0.750 represents a large amount and one in which it represents a small amount (e.g., 0.750 of a million dollars; 0.750 of a dollar).
- Using a thousandth grid, ask students to model the numbers 0.3, 0.30, and 0.300. Ask them to explain why the same area is shaded on each grid.
- Show students cards on which decimals have been written (e.g., 0.4 m, 0.75 m, and 0.265 m). Ask students to place the cards appropriately on a metre stick and to explain their thinking.
- Invite students to model 0.025 using a thousand grid. Then ask them to explain how this model differs from the model for 25 hundredths. Ask students to model 0.025 and 25 hundredths using base-ten blocks and to describe how the models are the same and how they are different.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- hundredths and thousandths grids
- hundredths circle
- number lines

MATHEMATICAL LANGUAGE

Teacher	Student
▪ decimals ▪ part of a set, part of a region, part of a unit of measure ▪ tenths, hundredths, thousandths, equivalent ▪ value	▪ decimals ▪ part of a set, part of a region, part of a unit of measure ▪ tenths, hundredths, thousandths, equivalent ▪ value

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 228–231, 234, 242–245
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 282–285, 288, 296–298
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 183, 184–185

Notes

SCO N09 Students will be expected to relate decimals to fractions and fractions to decimals (to thousandths). [CN, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N09.01 Express, orally and symbolically, a given fraction with a denominator of 10, 100, or 1000 as a decimal.
N09.02 Read decimals as fractions (e.g., 0.45 is read as zero and forty-five hundredths).
N09.03 Express, orally and symbolically, a given decimal in fraction form.
N09.04 Represent the fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$ as decimals using base-ten blocks, grids, and number lines.
N09.05 Express a given pictorial or concrete representation as a fraction or decimal (e.g., 250 shaded squares on a thousandth grid can be expressed as 0.250 or $\frac{250}{1000}$).

Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
N10 Students will be expected to relate decimals to fractions and fractions to decimals (to hundredths).	**N09** Students will be expected to relate decimals to fractions and fractions to decimals (to thousandths).	**N01** Students will be expected to demonstrate an understanding of place value for numbers greater than one million and less than one thousandth.

Background

Decimals are another way of writing fractions. Students should continue to build their conceptual understanding of the relationship of decimals to fractions as they explore numbers to the thousandths. Students should be asked to justify their thinking using concrete models.

One thousandth can be written as 0.001 or as $\frac{1}{1000}$. Students should be encouraged to read decimals as fractions (e.g., 0.246 is read as 246 thousandths and can be written as $\frac{246}{1000}$). Measurement contexts provide valuable learning experiences for decimal numbers because any measurement can be written in an equivalent unit that requires decimals (e.g., one metre is $\frac{1}{1000}$ of a kilometre; 1 m = 0.001 km).

To develop decimal and fractional number sense, it is essential to discuss the size of the number, such as 493 thousandths is about one half. Using number lines with benchmarks such as $\frac{1}{4}$ (0.25), $\frac{1}{2}$ (0.5), $\frac{3}{4}$ and (0.75) is helpful to create a visual reference for students.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

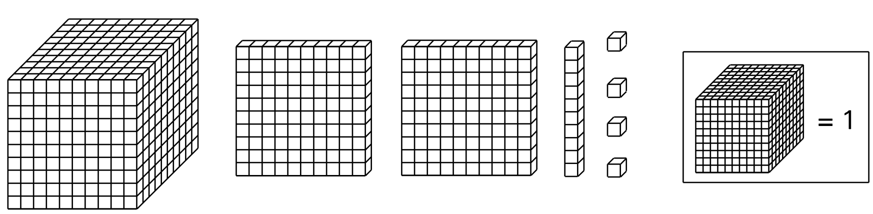
Tasks such as the following could be used to determine students' prior knowledge.

- Invite students to plot common fractions and decimal equivalents on a number line. For example: $\frac{5}{10}$ and 0.5; $\frac{25}{100}$ and 0.25; $\frac{8}{10}$ and 0.8; $\frac{1}{10}$ and 0.1. Ask them to explain why the fraction and the decimal are equivalent.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to model decimal thousandths using base-ten blocks.
  For example: 1.214



- Invite students to place decimals and fractions on a number line, such as: $\frac{3}{4}$, 0.31, $\frac{6}{10}$, $\frac{102}{1000}$.

- Tell students that they have properly placed 796 pieces of the 1000-piece jigsaw puzzle. Ask, What part (fractional and decimal) of the puzzle has been completed? What part of the puzzle has yet to be finished? ($\frac{204}{1000}$, 0.204)
- Invite students to model three decimal numbers using base-ten blocks or thousandths grids. Ask students to write the fraction equivalent for the decimal models they have created. Ask them to explain how they know that the fractions are correct.
- Present students with a pictorial representation of a number such as 1.031. Ask students to write as many fractions and decimals as they can for the picture.
- Ask students to explain why 0.750 is equivalent to $\frac{3}{4}$ using a metre stick or a number line.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to begin to explore the relationship between fraction and decimal benchmarks. For example, 0.5 is another name for $\frac{1}{2}$; 0.25 is another name for $\frac{1}{4}$; 0.75 is another name for $\frac{3}{4}$.
- Represent decimals in a variety of ways. For example: 0.452 is $\frac{452}{1000}$ and can be expressed as 0.4 + 0.05 + 0.002 or $\frac{4}{10} + \frac{5}{100} + \frac{2}{1000}$.

SUGGESTED LEARNING TASKS

- Invite students to express given numbers as fractions and decimals (e.g., sixty-four hundredths, $\frac{64}{100}$, 0.64).
- Ask students to investigate where in the media fractions and decimals are used and to write a report on their findings.
- Give students a “number of the day” and have them express this number in as many ways as they can. For example, 0.752 could be shown as: $\frac{752}{1000}$ or $\frac{7}{10} + \frac{5}{100} + \frac{2}{1000}$ or about $\frac{3}{4}$; plotted on a number line; modelled with base-ten materials on a place-value chart; shown on a thousandths grid; or described in a variety of ways (It’s 0.248 less than one whole, etc.).
- Invite students to play a concentration game in which they use a deck of cards, turned face down, some with decimals and some with fractions. Their task is to make matches by turning over two cards. They keeps the cards if the two cards are equivalent. They turn them back over face down if they do not match.
- Ask students to model three different decimal numbers using base-ten blocks or grids. Have students write the fraction equivalent for the decimal numbers they modelled and explain how they know their fractions are correct.
- Present students with a hundredth or thousandth grid that has been shaded to represent a decimal number. Invite students to write as many fractions and decimals as they can for the shaded area.
- Ask students to use a number line to explain why 0.750 is equivalent to $\frac{3}{4}$.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- number lines
- thousandths grids

MATHEMATICAL LANGUAGE

Teacher	Student
▪ equivalent ▪ hundredth and thousandth grid ▪ number lines ▪ relate decimals to fractions and fractions to decimals	▪ equivalent ▪ hundredth and thousandth grid ▪ number lines ▪ relate decimals to fractions and fractions to decimals

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 228–233
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 282–287
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 186–192
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 113–114

Notes

SCO N10 Students will be expected to compare and order decimals (to thousandths) by using benchmarks, place value, and equivalent decimals. [CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N10.01 Compare and order a given set of decimals by placing them on a number line that contains the benchmarks 0.0, 0.5, and 1.0.
N10.02 Compare and order a given set of decimals including only tenths using place value.
N10.03 Compare and order a given set of decimals including only hundredths using place value.
N10.04 Compare and order a given set of decimals including only thousandths using place value.
N10.05 Explain what is the same and what is different about 0.2, 0.20, and 0.200.
N10.06 Compare and order a given set of decimals, including tenths, hundredths, and thousandths, using equivalent decimals.

Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
N09 Students will be expected to describe and represent decimals (tenths and hundredths), concretely, pictorially, and symbolically. **N10** Students will be expected to relate decimals to fractions and fractions to decimals (to hundredths).	**N10** Students will be expected to compare and order decimals (to thousandths) by using benchmarks, place value, and equivalent decimals.	**N01** Students will be expected to demonstrate an understanding of place value for numbers greater than one million and less than one thousandth.

Background

Students should be able to determine which of two decimal numbers is greater by comparing the whole number parts first and then the amounts to the right of the decimal. It is important that students understand that decimal numbers do not need the same number of digits after the decimal to be compared. For example, it can be concluded that 0.8 > 0.423, without converting 0.8 to 0.800, because 0.8 is much more than one-half (a benchmark) and 0.423 is less than one-half. A common misconception is that students may think that 0.101 is greater than 0.11 because the whole number 101 is greater than the whole number 11. Others may think 0.101 is less than 0.11 because it has a digit in the thousandths place, while the other number has only hundredths. These same students may say 0.101 is less than 0.1 because it has thousandths while 0.1 has only tenths. Such misconceptions can be dealt with by having students create representations of the numbers that are being compared using models. It is helpful to use place value or equivalent decimals to compare and order these numbers. Students should be given opportunity to explore connections between models and oral and written forms. It is also beneficial to examine the connection made between decimals and fractions with denominators of 10, 100, and 1000 to understand **decimal equivalence** (e.g., 0.3 = $\frac{3}{10}$ or $\frac{30}{100}$ or $\frac{300}{1000}$).

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a number, such as 3.94 and ask them to
  - state the number that is 0.1 more than 3.94
  - record the numeral that is 1 less than 3.94
  - record the numeral that is 0.01 more than 3.94

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to compare and order decimal tenths, hundredths, and thousandths and express them as fractions. Ask them to explain how they know the order is correct.
- Ask students to continue the following series, counting by tenths.
  0.5, 0.6, 0.7, ____ , ____ , ____ , ____
- Ask students to demonstrate the equivalency of 0.5, 0.50 and 0.500 using base-ten blocks.
- Give students the number cards 0.99, 0.987, 0.9 and 1.001, and ask them which decimal number they think is closest to 1. Have them use words, pictures, or concrete materials to explain their thinking.
- Give students cards with various decimal numbers written on them, and ask students to place the numbers on a number line in appropriate places.

- Tell students that Michael says that 1.40 is greater than 1.406. Ask students to explain whether Michael is correct or not correct and to explain their thinking in words, pictures, or concrete materials.
- Tell students that gasoline is priced at 56.9 cents per litre. Ask students to explain what part of a dollar this is.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

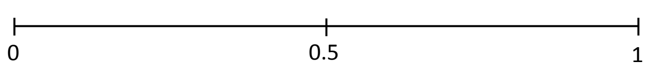
Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to place only decimal tenths on a number line, then repeat for decimal hundredths, and thousandths.



- Ask students to use a thousandth grid to model the equivalency of tenths, hundredths, and thousandths (e.g., 0.6, 0.60, 0.600) and explain what is the same and what is different about each model.
- Ask students to order a given set of decimals, including tenths, hundredths, and thousandths using equivalent decimals and to explain their thinking. For example, to order 0.402, 0.39, and 0.7, students could think of them as thousandths, 0.402, 0.390, 0.700.

- Students can be invited to make tables of distances (expressed in metres) that represents how far each person can hop on one foot, jump, kick a tissue, flip a coin, and other such tasks. They should then list each set of distances from least to greatest.

	Tissue	**Coin**	●●●
Joe	1.24 m	3.25 m	
Sarah	1.35 m	3.72 m	
Sergio	1.02 m	2.43 m	
●			
●			
●			

SUGGESTED LEARNING TASKS

- Give each student a different irregular shape and ask them to tear off about 0.256 of that shape. Have students explain how they estimated 0.256, and why pieces may not be the same size or shape.
- Provide students with a number line segment with end points labelled 2 and 4. Invite students to mark where they think each of the following numbers would be and to explain their thinking (2.3, 2.51, 2.999, 3.01, 3.75, 3.409, and 3.490).
- Give student partners six cards with different base-ten block pictures on them. Ask them to order the numbers represented and read the decimals to one another. Explain that the large cube represents 1 for this activity.
- Provide groups of students with a game board of the type shown below. Invite them to roll a number cube. On each roll of the number cube, students should fill in one blank on their game board with the number showing on the number cube. They will roll the number cube 18 times to enable them to fill in one number in each of the squares on the game board. The students who end up with three true number sentences, win a point. Repeat the process.

  □.□□ > □.□□
  □□.□ > □□.□
  □.□□ < □□.□

  Circulate during the play asking questions and noting students' justifications of position choice. Students could be asked to write any strategies they used on chart paper, and a class discussion could follow.
- Provide students with the number cards 9.023, 10.9, 9.05, 10.11, and 9.8. Ask them which decimal they think is closest to 10. Invite them to explain how they made their decision.
- Give students eight blank cards with a decimal number to the tenths on each. Have them challenge a partner to order the number cards. Repeat the activity using decimal numbers to the hundredths, and to the thousandths.
- Provide examples of some of the best javelin throw distances that have occurred in past Olympics (e.g., 1972: 90.48 m, 1980: 91.20 m, 1988: 84.28 m, 1992: 89.66 m). Ask students to arrange the distances in order and determine whether records always improve.
- Ask students to use pictures, words, and numbers to explain how 0.3, 0.30, and 0.300 are the same and how they are different.

- Ask students to use pictures, words, and numbers to explain how 0.25 and 0.250 are the same and how they are different.
- Divide students into groups of two or three. Each student creates a three-digit decimal number, less than 2. Next, they should build their number using base-ten materials and record their work with pictures. As a group, students place all numbers on a number line in relative position, and they compare and check answers with classmates.

Suggested Models and Manipulatives

- base-ten blocks
- number lines
- place-value charts
- thousandths grids

Mathematical Language

Teacher	Student
▪ benchmarks ▪ compare, order ▪ equivalent decimals ▪ number line ▪ place value ▪ tenths, hundredths, thousandths	▪ benchmarks ▪ compare, order ▪ equivalent decimals ▪ number line ▪ place value ▪ tenths, hundredths, thousandths

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 235, 243–244
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 288–289, 297–298
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 191–192

Notes

SCO N11 Students will be expected to demonstrate an understanding of addition and subtraction of decimals (limited to thousandths) [C, CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N11.01 Predict sums and differences of decimals using estimation strategies.
N11.02 Use estimation to correct errors of decimal point placements in sums and differences without using paper and pencil.
N11.03 Explain why keeping track of place-value positions is important when adding and subtracting decimals.
N11.04 Solve problems that involve addition and subtraction of decimals, limited to thousandths, using personal strategies.

Scope and Sequence

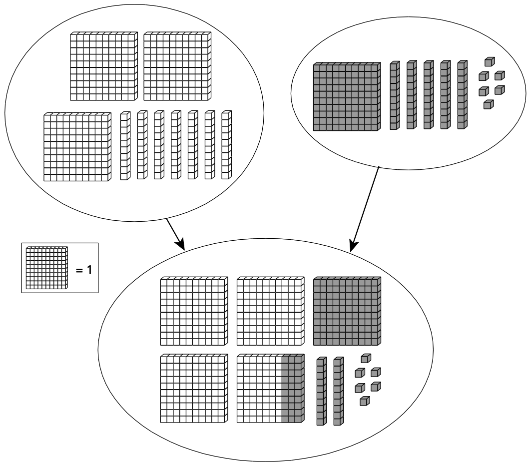
Mathematics 4	Mathematics 5	Mathematics 6
N11 Students will be expected to demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by ▪ estimating sums and differences ▪ using mental mathematics strategies to solve problems ▪ using personal strategies to determine sums and differences	**N11** Students will be expected to demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).	**N08** Students will be expected to demonstrate an understanding of multiplication and division of decimals (one-digit whole number multipliers and one-digit natural number divisors).

Background

For students to develop personal strategies for adding and subtracting decimal numbers, teachers may begin by asking them to solve addition and subtraction word problems involving money.

It is essential that students recognize that all of the properties and developed strategies for the addition and subtraction of whole numbers also apply to decimals. For example, adding or subtracting **tenths** (e.g., 3 tenths and 4 tenths are 7 tenths) is similar to adding or subtracting quantities of other items (e.g., 3 apples and 4 apples are 7 apples). This could be extended to addition with tenths that total more than one whole (e.g., 7 tenths and 4 tenths are 11 tenths or 1 and 1 tenth). The same is true with **hundredths** and **thousandths**. Rather than telling students to line up decimals vertically, or suggesting that they add zeroes, direct students to think about what each **digit** represents and what parts go together. For example, to add 1.625 and 0.34, a student might think using front-end addition, 1 whole, 9 (6 + 3) tenths, and 6 (2 + 4) hundredths, and 5 thousandths or 1.965.

Base-ten blocks and hundredths grids are useful models to represent addition with decimals up to hundredths. If a flat represents one whole unit, then 3.7 + 1.56 would be modelled as shown below and the sum would be found to be 5.26.



Students need to recognize that **estimation** is a useful skill when doing computations of whole and decimal numbers. Estimation can be used to determine if the sum or difference is reasonable and whether the decimal placement is correct. To be efficient when estimating **sums** and **differences** mentally, students need a variety of strategies from which to choose so they can select one that is efficient for the numbers involved. For example, a student may use front-end estimation to add 9.35 + 8.106. They would round each decimal to the nearest whole number (9 + 8) and know that the sum is greater than 17. Ensure that students have sufficient practice with a variety of mental mathematics and estimation strategies so that these acquired skills can be readily applied to solve various problems. When a problem requires an exact answer, students should first look at the numbers to determine if they are able to calculate the answer mentally. If no mental mathematics strategy can efficiently determine the answer for the numbers involved, then the student can explore which other paper-and-pencil strategy is best to use.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

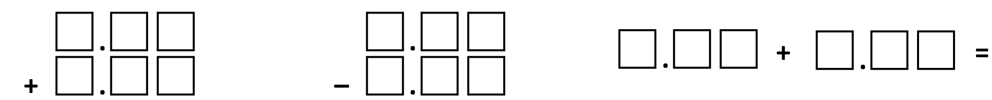
Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to find the difference for 2.3 – 1.8 or other similar computations and explain how they got their answer.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask the students to fill in the boxes so that the answer for each question is 0.4. The only restriction is that the digit 0 cannot be used to the right of the decimal points.



- Present the following situation in which Juli made an error when she subtracted. Ask students what could be said or shown to Juli to help her understand why the answer is incorrect.
  5.23 – 1.453 = 3.783.
- Ask students to use a model to explain how to find the sum and difference of two decimal numbers.
- Provide students with addition and subtraction questions in which the decimal is missing from the sum or difference. Have students place the decimal in the correct position in each answer.
- Tell students that Tim added 2.542 + 13.6 and said that the sum was 16.142. Roberto added the same numbers and said the answer was 2.678. Explain why the answers are different. Whose solution is correct? How do you know?
- Use an example to explain why it is important to keep track of place-value positions when adding and subtracting decimal numbers. (This can be written in a journal.)
- Invite students to solve problems such as the following:
  - Michel needs 2 kg of hamburger for a recipe. He has a 0.750 kg package. How much more does he need to buy?
  - Sasha bought two books at the book fair. One was $6.95 and the other was $7.38. How much change will she get from a $20 bill?
  - Maria's mathematics book has a mass of 0.573 kg, her social studies book has a mass of 0.45 kg, and her science book has a mass of 0.108 kg. What is the total mass of Maria's books?
- Ask students to use base-ten blocks to model, illustrate, and solve the following expressions:
  - 3.62 + 4.51
  - 3.21 + 1.4
  - 3.234 + 1.123
  - 1.59 + 1.238
  - 3.056 – 1.24
  - 1.234 – 0.8
- Model 2.13 and 1.291 with thousandth grids for students. Ask students to use the grids to explain how to find the sum of the two numbers.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide opportunities for students to model and solve addition and subtraction questions involving tenths, hundredths, and thousandths concretely, pictorially, and symbolically (e.g., thousandths and hundredths grids, base-ten blocks, and number lines).
- Present addition and subtraction questions both horizontally and vertically to encourage alternative computational strategies. For example, for 1.234 + 1.990, students might calculate 1.234 + 2 = 3.234 followed by 3.234 – 0.01 = 3.224.
- Ask students to investigate the relationship between adding decimals numbers and adding whole numbers. For example, 356 + 232 = 588; this looks similar to 0.356 + 0.232 = 0.588.
- Provide problem-solving situations that require students to add or subtract decimals using a variety of strategies.
- Ask students to determine an estimate when solving problems involving the addition and/or subtraction of decimals.
- Ask students to create and solve their own and each other’s word problems in a context that is relevant to them.
- Ensure that students estimate as part of the problem-solving process and explain their thinking.

SUGGESTED LEARNING TASKS

- Provide base-ten blocks or thousandth grids. Give the student addition or subtraction questions (decimal numbers) to represent using models. Be sure to include questions that require regrouping.
- Model 4.23 and 1.359 with base-ten blocks or thousandth grids. Ask students to use the materials to explain how to find the difference between the two numbers.
- Provide students with the batting averages of some baseball players. Ask them to calculate the spread between the player with the highest average and the one with the lowest. Invite students to create problems using the averages on the list.
- Request that the students provide examples of questions in which two decimal numbers are added and the sums are whole numbers.
- Tell students that you have added three numbers, each less than 1, and the result is 2.4. Ask if all the decimal numbers could be less than one half and to explain why or why not. Once students realize the numbers cannot all be less than one half, ask them how many could be less.
- Ask students to find situations in which decimals are added and subtracted beyond their classroom experiences and present their findings to the class.
- Provide students with problem-solving contexts such as those listed below. Ask them to estimate the sums or differences and then calculate the answer to the problem. Ask them to compare their calculated answer with the estimated answer.
  - Mount Everest is 8.850 km high. Mount Logan is 5.959 km high. What is their approximate height difference?
  - Keisha and her 11 friends go out to eat at a restaurant. Each meal costs $9.97 including taxes and the tip. Will $100 cover the cost of the meals?
  - You have a piece of string and cut it off at 46.8 cm, leaving 138.6 cm. Estimate the length of string you had at the beginning.
  - Adam was given two Newfoundland puppies, named Ebony and Ireland, for his birthday. When they were born, Ebony had a mass of 0.775 kg and Ireland had a mass 0.836 kg. Estimate the total mass of Ebony and Ireland and explain your estimation strategy.
- Invite students to create a problem that requires only an estimated answer to solve it. Then, ask them to solve the problem they created by estimating the answer and explain their thinking.
- Ask students to explain if a $10 bill would cover the cost of buying a milkshake for $3.98 and a salad for $6.59.
- Have students explain how to estimate that amount of money needed if Jimmy is going to buy three packages of gum and each package of gum costs $1.37.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- hundredth and thousandth grids
- number lines

MATHEMATICAL LANGUAGE

Teacher	Student
▪ estimating sums and differences ▪ exact and approximate ▪ place value ▪ tenths, hundredths, thousandths	▪ estimating sums and differences ▪ exact and approximate ▪ place value ▪ tenths, hundredths, thousandths

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 236–238
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 290–292
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 197–198
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 125–126

Notes

Patterns and Relations (PR)

GCO: Students will be expected to use patterns to describe the world and solve problems.

GCO: Students will be expected to represent algebraic expressions in multiple ways.

SCO PR01 Students will be expected to determine the pattern rule to make predictions about subsequent terms (elements). [C, CN, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR01.01 Extend a given increasing or decreasing pattern, with and without concrete materials, and explain how each term differs from the preceding one.
PR01.02 Describe, orally or in written form, a given pattern using mathematical language such as **one more**, **one less**, or **five more**.
PR01.03 Write a mathematical expression to represent a given pattern, such as $r + 1$, $r - 1$, $r + 5$.
PR01.04 Describe the relationship in a given table or chart using a mathematical expression.
PR01.05 Determine and explain why a given number is or is not the next term in a pattern.
PR01.06 Predict subsequent terms in a given pattern.
PR01.07 Solve a given problem by using a pattern rule to determine subsequent terms.
PR01.08 Represent a given pattern visually to verify predictions.

Scope and Sequence

Mathematics 4	**Mathematics 5**	**Mathematics 6**
PR01 Students will be expected to identify and describe patterns found in tables and charts, including a multiplication chart.	**PR01** Students will be expected to determine the pattern rule to make predictions about subsequent terms (elements).	**PR01** Students will be expected to demonstrate an understanding of the relationship within tables of values to solve problems. **PR02** Students will be expected to represent and describe patterns and relationships, using graphs and tables.

Background

“Patterns are key factors in understanding mathematical concepts. The ability to create, recognize, and extend patterns is essential for making generalizations, seeing relationships, and understanding the order and logic of mathematics.” (Burns 2007, 144) These skills provide the groundwork for algebraic reasoning and inquiry.

Patterns represent identified regularities based on rules describing the patterns’ terms. Unless a pattern rule is provided there is no single way to extend a pattern. For example, 1, 3, 5, 7 might be an odd number sequence, or it might be a repeating sequence 1, 3, 5, 7, 1, 3, 5, 7, … In Mathematics 5, students build on previous knowledge of increasing and decreasing patterns and focus on describing these patterns and their relationships mathematically. They will use this knowledge to make and verify predictions of missing terms in various patterns.

The elements that make up increasing and decreasing patterns are called **terms**. Each term builds on the previous term. Using a table to represent an increasing or decreasing pattern can help students organize their thinking. It can also help them generalize the patterns symbolically (create a rule). A pattern rule must describe how each and every term of the pattern is generated, including the first term. For example, 2, 5, 8, 11, ..., can be described as "start at two and add three each time."

Patterns can be used to represent a situation and to solve problems. They can be **extended** with and without concrete materials and can be described using mathematical language. When discussing a pattern, students should be encouraged to determine how each term in the pattern is different from the preceding term. They should ask themselves, What remains constant? What is changing?

Writing a pattern rule for an increasing pattern, such as that shown below, should describe how each and every term of the pattern is generated and should include the starting point. For example, the pattern rule below could be described as "start at three, add three each time."

xxx	xxx	xxx	xxx
	xxx	xxx	xxx
		xxx	xxx
			xxx
3	6	9	12

In describing the pattern, students observe that the number of Xs increases by three each time. This is recursive thinking and tells how to find the value of a term given the value of the preceding term. Students are expected to also see that they can determine the number of Xs by multiplying the term number by three. This is functional thinking focused on the relationship between the term and the term value.

Tables and charts provide an opportunity to display patterns, see relationships, and develop functional thinking. For most students, these tables and charts make it easier to see the patterns from one term to the next. When a chart has been constructed, the differences from one term to the next can be written by it. Students will probably first observe the pattern from one term to the next (recursive thinking). However, using the chart or a recursive pattern to find the twentieth or hundredth step is not reasonable or efficient. If a rule or relationship can be discovered (functional thinking), any table entry can be determined without building or calculating all of the intermediate entries.

Term #	1	2	3	4	5	6	?	...	20
Number of Xs	3	6	9	12	?	?	?	...	?

Students will learn that the rule can be described as a **mathematical expression**. For example, in the above pattern, the rule could be described as $3 \times n$ or $3n$. Therefore the 20th term would be 3×20, which is 60. Students should be given frequent opportunity to use materials to represent patterns and explain orally and in writing how terms in various patterns change as the patterns are extended.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a multiplication grid. Ask them to describe some of the patterns they observe.

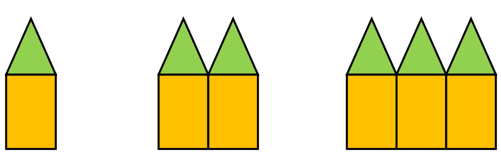
Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Invite students to fill in missing terms from number sequences and identify the pattern rules.
  (a) 1, 4, _____, 16, ___, 36
  (b) 18, 16, 14, _____, _____
  (c) 2.4, 2.7, _____ , _____ , 3.6
- Give students a chart showing the input and the output and ask them to provide a possible rule.

Input	Output
2	9
3	10
4	11
5	12

- Show students the picture below. The first term is made up of two pattern blocks. The second term is made up of four pattern blocks, and the third term is made up of six pattern blocks. Have students predict the number of pattern blocks in the fourth term and the eighth term. Ask them to use pattern blocks or draw a picture of each of the eight terms to verify their answer.



- Ask students to tell whether 84 would be included in each of the following patterns and to explain how they know:
  (a) 1, 3, 5, 7, …
  (b) 4, 8, 12, 16, …
  (c) 200, 192, 184, 176, …
- Ask students to solve real-world problems that require them to identify a pattern rule to determine subsequent terms. For example, to bake cookies for a school bake sale, the quantities of ingredients in the recipe must be determined for multiple batches of cookies. If two cups of sugar and three cups flour are needed for one batch, how much is needed for four batches? seven batches?
- Show students a table that shows the relationship between the number of students going to a movie and the total cost of the tickets. Ask students to describe the relationship between the students and the cost of the tickets using a mathematical expression. Use the pattern to determine the number of students at the movie if the tickets cost $98.

Students	1	2	3	4	?
Cost of tickets	$7	$14	$21	$28	$98

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

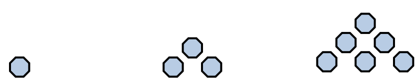
Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Invite students to practise extending patterns with materials and drawings and then translate the pattern to a table or T-chart. Ask them to describe what is happening as the pattern increases or decreases and how the new term is related to the previous one.
- Ask students to describe, using mathematical language (e.g., one more, seven less) and symbolically (e.g., $r + 1$, $p - 7$), a pattern represented concretely, pictorially, or from a chart.
- Ask students to verify whether or not a particular number belongs to a given pattern.
- Provide students with opportunities to predict terms in a pattern. They should be expected to explain and verify their predictions. Visual representations, such as models or drawings are helpful.
- Ask students to solve problems and make decisions based upon the analysis of a pattern.

Suggested Learning Tasks

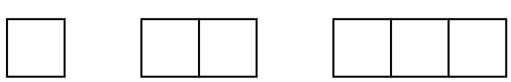
- Show students the first three or four steps of a pattern. Provide them with appropriate models and grid paper and have them extend the patterns recording each step, and explain why their extension follows the pattern. Have them determine a pattern rule.



- Ask students to examine number sequences to determine subsequent terms and explain their extensions. Ask students to determine the pattern rule.

  1, 4, 7, 10, 13, …   42, 36, 30, 24, 18, …   0, 2, 6, 14, 30, …

  For each sequence, ask students to give two numbers that cannot come next and to explain why.
- Ask students to work in pairs to explore the many patterns on a multiplication chart (e.g., square numbers on the diagonal, sums of rows and columns, adjacent square patterns, doubling between columns, such as the 2s, 4s, and 8s).
- Provide students with an increasing pattern and have them extend it. They should make a table showing how many items are needed to make each step of the pattern. Have them predict the number of items in the tenth or twentieth step of the pattern. For example, four people can sit at one table, six people can sit at two tables pushed together, or eight people can sit at three tables. How many can sit at ten tables? Twenty? How many tables are needed for 24 people?



Number of tables	1	2	3	4	…
Number of seats	4	6	8	?	…

This pattern could be displayed on a T-chart

Tables	Seats

- Present students with a problem involving a pattern and ask them to solve it.
  - Suzette delivers pizza. Each day, she earns $20. How much will she earn at the end of one day? How much will she earn at the end of two days? How much will she earn at the end of one week? How many days will it take Suzette to earn $240?
    Ask students to explain how they solved the problem.
  - All books in the Scholastic flyer are on sale for $15 each. How much would it cost to buy two books? Three books? Four books? Create a table to find the cost. Describe the pattern rule. How much would it cost to buy nine books? How do you know?
- Ask students to match the pattern to its variable expression.

Variable Expressions	Patterns
(a) $4 + n$	4, 5, 6, ...
(b) $n + 3$	5, 6, 7, ...
(c) $17 - n$	14, 13, 12, ...
(d) $15 - n$	16, 15, 14, ...

- The table shows the relationship between the number of students on a field trip and the cost of providing boxed lunches.

Customers	1	2	3	4	5
Cost of lunch in dollars	$3	$6	$9	$12	$15

  Ask students to explain how the cost of lunches is related to the number of students. Then, ask students to explain the pattern they observe to show the relationship between the number of students and the cost of lunches. Finally, ask students to use the pattern to help determine the number of students on the bus if the lunches cost 90 dollars in total?

Suggested Models and Manipulatives

- calculator
- colour tiles
- counters
- dot paper
- grid paper
- linking cubes
- multiplication chart
- pattern blocks

Mathematical Language

Teacher	Student
▪ expression	▪ expression
▪ extending patterns	▪ extending pattern
▪ increasing and decreasing patterns	▪ increasing and decreasing patterns
▪ one more, one less, five more	▪ one more, one less, five more
▪ pattern rule	▪ pattern rule
▪ predict	▪ predict
▪ tables and charts	▪ tables and charts
▪ term	▪ terms

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 570–571, 572, 573–577
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 608–609, 610–611, 612–616
- *Teaching Student-Centered Mathematics, Grades 3–5*, (Van de Walle and Lovin 2006), pp. 293–296, 306–307, 318–319
- *Teaching Student-Centered Mathematics, Grades 5–8*, (Van de Walle and Lovin 2006), pp. 265–267

Video

- *Analyzing Patterns (Skip Counting) on a Hundred Board* (27:16 min.) (ORIGO Education 2010)

Notes

SCO PR02 Students will be expected to solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions. [C, CN, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR02.01 Explain the purpose of the letter variable in a given addition, subtraction, multiplication, or division equation with one unknown (e.g., $36 \div n = 6$).
PR02.02 Express a given pictorial or concrete representation of an equation in symbolic form.
PR02.03 Express a given problem as an equation where the unknown is represented by a letter variable.
PR02.04 Create a problem for a given equation with one unknown.
PR02.05 Solve a given single-variable equation with the unknown in any of the terms (e.g., $n + 2 = 5$, $4 + a = 7$, $6 = r - 2$, $10 = 2c$, $15 \div r = 3$).
PR02.06 Identify the unknown in a problem; represent the problem with an equation; and solve the problem concretely, pictorially, or symbolically.

Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
PR05 Students will be expected to express a given problem as an equation in which a symbol is used to represent an unknown number. **PR06** Students will be expected to solve one-step equations involving a symbol to represent an unknown number.	**PR02** Students will be expected to solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.	**PR03** Students will be expected to represent generalizations arising from number relationships using equations with letter variables. **PR04** Students will be expected to demonstrate and explain the meaning of preservation of equality concretely, pictorially, and symbolically.

Background

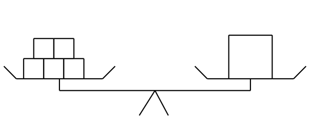
Algebra is a system that allows us to represent and explain mathematical relationships. Exploring patterns leads to algebraic thinking. Students are thinking algebraically when they solve open number sentences like 5 + ☐ = 13, first using boxes or open frames, then using letters, $5 + n = 13$. Students usually progress from the use of open frame to letters. When letters are used in mathematics, they are called variables. It is useful for students to think of variables as numbers that can be operated on and manipulated like other numbers.

Through the use of balance scales and concrete representations of equations, students will see the equal sign means “is the same as,” and acts as the midpoint or balance, with the quantity on the left of the equal sign being the same as the quantity on the right. When the quantities balance, there is **equality**. When there is an imbalance, there is **inequality**.

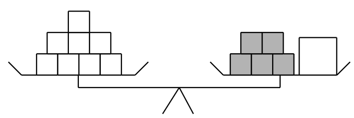
Students need to be able to write a problem for a given equation such as $n \div 7 = 16$. The word problem must make sense of the equation. For the above equation a student might write, There are sixteen weeks until we have holidays. How many days are there before holidays begin? Students are also asked to write an equation for a situation such as, Kate is four times as old as Anna. If Kate is 12 years old, how old is Anna?

Students have been exploring the concept of equality since Mathematics 1. It is important for students to recognize that the **equal sign** indicates that both sides of the equation are balanced and does not mean “the answer is.” We need to continue to use language such as, “is the same as” or “is equal to” for the equal sign. An equation is a mathematical sentence with an equal sign.

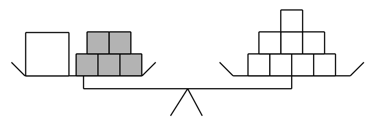
In order to solve an equation, we need to find the value of the variable that makes the equation a true statement, that is to determine the value of the variable that will balance both sides of the equation. Using the balance concept on a regular basis will help students develop a visual image for solving equations. Students should begin solving equations using balance scales and concrete materials, before moving to pictures and equations.



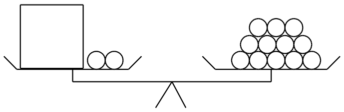
$5 = r$



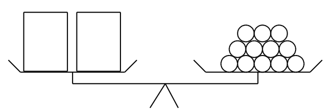
$8 = 5 + b$



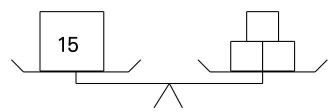
$r + 5 = 8$



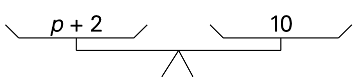
$r + 2 = 12$

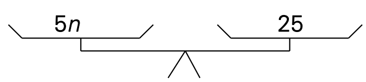


$2n = 12$



$15 \div 3 = n$





An **expression** does not include an equal sign and is used most frequently to describe a pattern rule. A **coefficient** in elementary algebra is the numerical part of an expression usually written before the literal part (letter). In the expression, $3b$, the 3 is the coefficient. A term is part of an algebraic equation or expression that may be a number, a variable, a product of both, or a quotient (where the variable is not in the denominator) of both. A term that is a number is called a **constant**.

$k + 6$, k is a term and 6 is a term.

$35 = 7y$; 7 is the coefficient in the term $7y$.

An unknown is “... a specific value yet to be determined. Sometimes there is only one possible value for the unknown, sometimes there are no possible values, and sometimes there are multiple values or an infinite number of values.” (Small 2008b, 582)

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

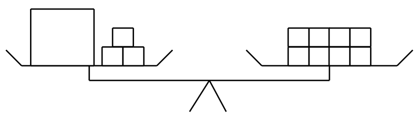
- Ask students to tell what number the box represents in the following equation: $15 - \square = 8$.
- Tell students that you have 24 marbles and your friend gives you some more marbles. Now you have 32 marbles in all. How many marbles did your friend give you?
  (a) Write an equation to show what is happening in this problem.
  (b) Solve the problem. Explain your thinking.

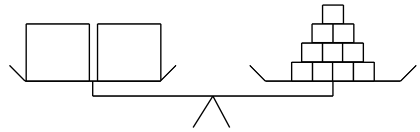
Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to draw a diagram and solve single-variable, one-step equations such as the following:
  $18 + n = 31$ $9 = 43 - p$ $8k = 56$ $m \div 6 = 7$
- Then, ask students to write word problems that could be represented by each of the above equations.
- Tell students that Nick was given the following problem to solve.
- There were some students on the bus and 12 more got on. Now there are 14 students on the bus. How many students were originally on the bus? To solve the problem, Nick wrote this equation $b + 12 = 14$. Ask students to explain why Nick used a letter in the equation.
- Ask students to write an equation for the following problem: Shamar placed 24 apples in a basket. His friends arrived and ate some of the apples from the basket. There are now 15 apples in a basket. How many apples did his friends eat?
- Ask students to write an addition, subtraction, multiplication, or division story problem. Then, ask them to write an equation with an unknown to represent their story problem. Have students solve the equation. Ask them to write another possible equation with an unknown that could be written for the same problem and explain why two equations are possible for the same story problem.

- Tell students that Marci said the *w* in the following equation 16 = *w* – 4 equals 12. Ask students to explain whether Marci is correct and to explain how they know.
- Ask students to write equations to describe the balance representations, such as the following:





- Ask students to draw a diagram to represent and solve the following equations.
  (a) $n + 12 = 19$
  (b) $k = 14 - 3$
  (c) $9 + d = 16$
- Ask students to solve the following equations and explain their thinking.
  (a) $c - 12 = 8$
  (b) $4 + 5 = v + 2$
  (c) $3n = 15$
  (d) $85 = r \div 5$
  (e) $21 + y = 40$
  (f) $24 = p + 9$
  (g) $25 = 35 - p$

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

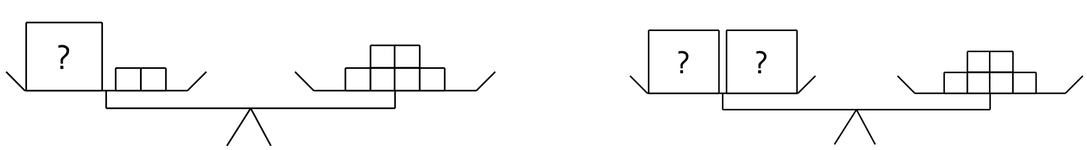
Consider the following strategies when planning daily lessons.

- Build on students' knowledge from previous grades to write addition, subtraction, multiplication, and division equations. Connect the concrete (use models such as counters and balance scales) with pictorial and symbolic representations consistently as the students develop and demonstrate an understanding of equations.
- Use everyday contexts for problems to which students can relate so that they can translate the meaning of the problem into an appropriate equation using a letter to represent the unknown number.
- Ask students to create problems that use a variety of number sentences and the four operations.
- Explain that if the same variable, or unknown, is used repeatedly in the same equation, then there is only one possible solution for that variable or unknown (e.g., for $n + n = 20$ can be written as $2n = 20$).
- Provide one-step single-variable equations and have students create story problems.
- Ask students to complete tables such as the one below.

n	**3*n***
3	
8	
	30
12	

SUGGESTED LEARNING TASKS

- Ask students to play, "Solve for my variable."
  - (a) I subtract 6 from *n* and have 13 left. What is *n*?
  - (b) Four more than *p* is 37. What is *p*?
  - (c) Two more than *w* is 23. What is *w*?
  - (d) One less than *k* is 27. What is *k*?
  - (e) Two times a number (*p*) is 14. What is *p*?
  - (f) Half of *r* is 6. What is *r*?
- Provide simple story problems and ask students to write equations that represent the problems. Include stories for all four operations. For example,
  - (a) I had birthday money and spent \$6.25. I now have \$8.75 (n – \$6.25 = \$8.75 or \$6.25 + \$8.75 = n).
  - (b) There are three full boxes of pencils. There are 36 pencils in all ($3a = 36$ or $a + a + a = 36$).
- Ask students to write equations for balances such as the ones below, using letters for the variables.



- Provide students with problems such as Fran is three years older than Hannah. Hannah is 21 years old. How old is Fran? Ask students to write an equation to solve the problem and then solve the equation. Ask students to explain whether it is possible to write a different equation for the same problem and to explain their thinking.
- Tell students that Max said that p in the following equation, $p - 8 = 14$, equals 6. Ask students to tell whether Max is correct and to explain their thinking.
- Present students with the following problem: There are now 11 muffins on a tray. There were 24 at the start. Some have been eaten. How many muffins are missing from the tray?
  - Ask students to write an equation to represent the problem, and then solve the equation.
  - Then ask students whether there was another possible equation that they could have written for the same problem? Ask them to explain their thinking.
- Invite students to use concrete materials such as blocks or counters and the balance scales to find the value of p in each of the following equations. If necessary, model the use of guess and test as one strategy. By observing patterns in their results, students become more systematic in the guesses they make.

  $3 + p = 11$

  $14 - p = 8$

  $p - 9 = 16$

  $p + 5 = 17$

Suggested Models and Manipulatives

- balance scales
- counters
- linking cubes

Mathematical Language

Teacher	Student
▪ letter variable in addition, subtraction, multiplication, and division equation ▪ number coefficients ▪ solve one-step equations ▪ symbol ▪ unknown number	▪ letter variable in addition, subtraction, multiplication, and division equation ▪ numerical factor, which is not a variable ▪ solve one-step equations ▪ symbol ▪ unknown number

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), p. 582, 585–588
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 620–621, 624–627
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 306–307

Notes

Measurement (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

SCO M01 Students will be expected to design and construct different rectangles, given a perimeter or an area or both (whole numbers), and make generalizations. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

M01.01 Draw two or more rectangles for a given perimeter in a problem-solving context.
M01.02 Draw two or more rectangles for a given area in a problem-solving context.
M01.03 Determine the shape that will result in the greatest area for any given perimeter.
M01.04 Determine the shape that will result in the least area for any given perimeter.
M01.05 Provide a real-life context for when it is important to consider the relationship between area and perimeter.

Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
M03 Students will be expected to demonstrate an understanding of area of regular and irregular 2-D shapes by ▪ recognizing that area is measured in square units ▪ selecting and justifying referents for the units square centimetre (cm^2) or square metre (m^2) ▪ estimating area using referents for cm^2 or m^2 ▪ determining and recording area (cm^2 or m^2) ▪ constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area. **Note:** Perimeter is addressed in Mathematics 3.	**M01** Students will be expected to design and construct different rectangles, given a perimeter or an area or both (whole numbers), and make generalizations.	**M03** Students will be expected to develop and apply a formula for determining the ▪ perimeter of polygons ▪ area of rectangles ▪ volume of right rectangular prisms

Background

In Mathematics 4, students worked extensively with area, including finding the area and constructing different rectangles for a given area. Students should be able to explain that a square centimetre is a measure of area using a square that is one centimetre on each side. Students should connect this with the area of a face on a metric linking cube or the small cube from the base-ten materials. Students also built a square metre and observed its relationship to the square centimetre. Perimeter was addressed in Mathematics 3 but not in Mathematics 4. Students may require a review of perimeter before they begin constructing shapes given a specific perimeter. This year the focus will be on working with area and perimeter when constructing rectangles. Students will be required to make conclusions regarding rectangular shapes that are created to make the greatest or least areas.

Since the focus of the curriculum is teaching through problem solving, concepts of perimeter and area will be presented in a real-world problem-solving context. This outcome could be introduced by asking students to solve a problem such as the following: Zack has 20 pieces of interlocking fence, each one metre long. How many different rectangular-shaped enclosures can he make using all the pieces of fence? What shape should he choose if he uses the fenced area for his dog? What shape might he use if he uses the fenced area for a garden?

Students in Mathematics 5 often do not make the distinction between area and perimeter and when to use each one in a problem. They may calculate the area instead of the perimeter or vice versa. **Area** is the measure of the space inside a region or how much it takes to cover a region. **Perimeter** is the distance around a region. It is important that students have many opportunities to construct rectangles of different areas and perimeters concretely and pictorially. Students should recognize that area and perimeter are independent of one another.

Area and perimeter involve measuring length. Rules or formulas may be invented by students as they do the activities, but formal instruction related to formulas will take place in Mathematics 6. When students are able to measure efficiently and effectively using standard units, their learning experiences can be directed to situations that encourage them to construct measurement formulas. When determining the area of a rectangle, students may realize as they count squares that it would be quicker to find the number of squares in one row and multiply this by the number of rows. When finding perimeters of rectangles, students may discover more efficient methods instead of adding all four sides to find the answer (e.g., add the length and width and double the sum).

It is important that students learn about area and perimeter together. Through explorations, students will discover that

- it is possible for rectangles of a certain area to have different perimeters
- it is possible for rectangles with the same perimeter to have different areas
- the closer the shape is to a square, the larger the area will be
- for any given perimeter, the rectangle with the smallest possible width will result in the least area

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

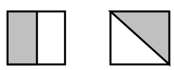
Guiding Questions

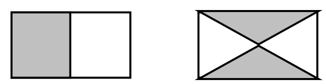
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to estimate the area for each of the following pairs of congruent shapes. Then, they should decide if the shaded part has the same area in each pair of shapes. Ask students to explain their thinking.

(a) 

(b) 

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to create at least two different rectangles on a geo-board with a perimeter of 12 units. Have students explain how they decided on the dimensions for the rectangles. Ask students to determine if all of the rectangles have the same area and to explain their thinking.
- Provide students with grid paper and invite them to draw at least two different rectangles with an area of 24 square centimetres. Ask students to determine if all of the rectangles have the same perimeter and to explain their thinking.
- Have students compare and contrast a given pair of rectangles, each having the same perimeter. Ask them to consider the following questions: How is a rectangle with dimensions of 3 cm × 4 cm different than a rectangle with dimensions of 2 cm × 5 cm? How are they similar?
- Ask students to choose the dimensions of the rectangle with the largest area and the smallest area from a set of rectangles with the same perimeter. For example, present students with the following problem: The following rectangles all have a perimeter of 18 cm: (1 cm × 8 cm), (2 cm × 7 cm), (3 cm × 6 cm), and (4 cm × 5 cm). Which of these rectangles has the largest area? Which has the smallest area? Explain your thinking.
- Invite students to construct (concretely or pictorially) two or more rectangles with a specified perimeter and to record the dimensions of each rectangle. Ask students to select and justify dimensions that would be most appropriate in a particular situation. For example, if a rectangle is to have a perimeter of 36 m, what are the dimensions of the possible rectangles? Which rectangle would be most appropriate if the rectangle is to be the floor of a room? The base of a dog pen? A vegetable garden?
- Ask students to construct (concretely or pictorially) and record the dimensions of as many rectangles as possible with a specified area. Ask them to select, with justification, the rectangle that would be most appropriate in a particular situation. For example, a rectangle is to have an area of 24 square units, what are the dimensions of the possible rectangles? Which rectangle would be most appropriate if the rectangle is meant to be the largest flower bed possible? The smallest flower bed possible?

- Have students identify situations relevant to self, family, or community where the solution to problems would require the consideration of both area and perimeter, and solve the problems. For example, a farmer has 22 metres of fencing to build a rectangular chicken coop. Which dimensions would provide the largest area for the chickens?

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to use geo-boards to construct rectangles with specified perimeters and discuss the areas.
- Assign specific areas (e.g., 12 square units) and have students use colour tiles to create various rectangles and find the possible perimeters.
- Provide students with a variety of real-world contexts in which to explore the relationships between area and perimeter (e.g., flooring, fencing, playing fields, gardens, zoo enclosures, tennis courts, wallpaper, bowling alleys).
- Solve a variety of contextual problems related to area and perimeter. For example, the cast of the latest hit movie is coming to your town. Design a “red carpet” for the event with an area of 50 m^2 that will provide the maximum amount of room for photographers and fans to stand around its perimeter.

SUGGESTED LEARNING TASKS

- Ask students to use dot paper to compare the areas of rectangles with the following dimensions: a length of 2 cm and a width of 3 cm, a length of 4 cm and a width of 3 cm, a length of 6 cm and a width of 3 cm. Ask students to describe what they observe. Ask them to suggest another set of dimensions that follows the same pattern and to draw conclusions.

- Have students explain why the perimeter of rectangles with whole number side lengths is always even. Invite them to use words, drawings, and/or numbers in their explanation.
- Ask students to relate perimeters to areas. For example, give pairs of students 24 colour tiles and invite them to create different rectangles, each with the area of 24 square units, but with different perimeters. Ask them to find a way to keep track of their rectangles and perimeters. Ask them to explain which rectangle has the largest perimeter. The smallest? Invite students to draw conclusions about the dimensions of the rectangle and the resulting perimeter.
- Ask students to draw three different rectangles, each having the same perimeter.
- Invite students to construct, on square grid paper, rectangles with a given perimeter. Ask them to compare the side lengths and the areas. Have students discuss their findings and conclusions with regard to side length and area.
- Provide a 2 × 3 rectangle. Have students predict what would happen to the area and perimeter if the side lengths were doubled or halved. Invite students to check their predictions and draw conclusions based on their investigation.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- dot paper
- geo-boards
- grid paper
- linking cubes

MATHEMATICAL LANGUAGE

Teacher	Student
▪ design, construct ▪ measure, estimate ▪ metre or centimetre; square metre or square centimetre ▪ relationship between area and perimeter	▪ design, construct ▪ measure, estimate ▪ metre or centimetre; square metre or square centimetre ▪ relationship between area and perimeter

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 388–393, 402–403
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 434–439, 448
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 264–265, 288–289

Notes

SCO M02 Students will be expected to demonstrate an understanding of measuring length (mm) by ▪ selecting and justifying referents for the unit millimetre (mm) ▪ modelling and describing the relationship between millimetre (mm) and centimetre (cm) units, and between millimetre (mm) and metre (m) units [C, CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

M02.01 Provide a referent for one millimetre, and explain the choice.
M02.02 Provide a referent for one centimetre, and explain the choice.
M02.03 Provide a referent for one metre, and explain the choice.
M02.04 Show that 10 millimetres is equivalent to one centimetre, using concrete materials.
M02.05 Show that 1000 millimetres is equivalent to one metre, using concrete materials.
M02.06 Provide examples of instances where millimetres are used as the unit of measure.
M02.07 Estimate and measure length in millimetres, centimetres, and metres.

Scope and Sequence

Mathematics 4	**Mathematics 5**	**Mathematics 6**
M03 Students will be expected to demonstrate an understanding of area of regular and irregular 2-D shapes by ▪ recognizing that area is measured in square units ▪ selecting and justifying referents for the units square centimetre (cm^2) or square metre (m^2) ▪ estimating area using referents for cm^2 or m^2 ▪ determining and recording area (cm^2 or m^2) ▪ constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area. **Note:** Perimeter is addressed in Mathematics 3.	**M02** Students will be expected to demonstrate an understanding of measuring length (mm) by ▪ selecting and justifying referents for the unit millimetre (mm) ▪ modelling and describing the relationship between millimetre (mm) and centimetre (cm) units, and between millimetre (mm) and metre (m) units	**M03** Students will be expected to develop and apply a formula for determining the ▪ perimeter of polygons ▪ area of rectangles ▪ volume of right rectangular prisms

Background

Measurement is fundamentally about making comparisons. At this point in their learning, students are able to compare two objects directly by accurately using standard units of length such as **centimetres** and **metres**. In Mathematics 5, students will extend this knowledge to include **millimetres**.

Students are expected to have a **personal referent** for one millimetre and be able to explain their choice. They should continue to use their referents for one centimetre and one metre developed in Mathematics 3. Examples of referents are as follows: one millimetre is about the thickness of a dime, one centimetre is about the width of your baby finger, and one metre is about the height of the doorknob from the floor.

In Mathematics 3, students have already explored the relationship between centimetres and metres, and should now explore the relationship between centimetres and millimetres and millimetres and metres. Students should recognize that one metre is 100 centimetres, one metre is 1000 millimetres, and one centimetre is 10 millimetres.

Students need to learn how to choose the appropriate unit or combination of units for the task at hand. This choice depends on the magnitude of the length to be measured and the level of precision required by the task (Small 2008b, 379). For example, millimetres can be used to measure small objects or to measure larger objects with more precision. Flexibility with using the different measurements is in the developmental stage and needs to be supported with a variety of materials and many experiences. Students need to be able to rename measurements and change from smaller units to larger units and vice versa, but also to be able to identify which unit is the most appropriate. For example, a pencil that is 11 cm long could also be described as 110 mm.

It is important that students are encouraged to estimate measurements before actually verifying them using a measurement tool. Using rulers, metre sticks, Cuisenaire rods, and base-ten blocks will provide students with benchmarks when estimating lengths.

Teachers need to assist students with using rulers accurately. Often students will fail to consider the gap between the end of the ruler and the zero mark. Some students may start at 1 on the ruler and still use the ruler accurately by taking this into consideration.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to estimate the area of a rectangle and explain what referent they used.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Invite students to provide examples of what they would measure in millimetres. Ask students to explain why they would use this unit and why these units are useful.
- Ask students to provide examples of situations that are relevant to their life, family, or community in which linear measurements would be made and identify the standard unit (mm, cm, m) that would be used for that measurement. Ask students to justify their choice (e.g., heights of people, length of the bus).
- Ask students to use metric units to fill in the blanks below in as many ways as possible.
  10 ____ = 1 ____ or 100 ____ = 1 ____ or 1000 ____ = 1 ____ .
- Invite students to draw, construct, or physically act out a representation of a given linear measurement.
- Ask students to pose and solve problems that involve hands-on linear measurements using either referents or standard units.
- Tell students that a grasshopper hopped 2000, 1000, and 1500 mm. Ask them to write these distances in metres.
- Invite students to look around the classroom, choose one object, and estimate its measurement. Ask students to identify the referent they used to determine their estimate and to explain their thinking.
- Ask students to measure the length of their desks in centimetres. Then have them measure in millimetres. Ask students to explain which unit was most appropriate and why, and which unit is most precise and why.
- Have students identify something that would be measured in millimetres. In centimetres. Metres.
- Ask students to identify something that would not be measured in millimetres. In centimetres. Metres.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to generalize measurement relationships between millimetres, centimetres, and metres from explorations using concrete materials (e.g., 10 mm = 1 cm; 100 cm = 1 metre, 1000 mm = 1 m, 0.01 m = 1 cm, 0.001 m = 1 mm, 0.1 cm = 1 mm).
- Ask students to choose and use referents for 1 mm, 1 cm, and 1 m to determine approximate linear measurements in situations relevant to self, family, or community and explain the choice.
- Help students develop mental images of various measurement standards. To provide estimation practice, involve students in activities such as, Show me (with hands or arms) a length of about 75 centimetres; 20 millimetres; 0.5 metres; take a step that is about 1 m in length, 30 cm in length; hop forward on one foot a distance of 300 mm, 10 cm, about 0.5 m.
- Ask students to use the relationships between standard metric units to rename measurements when comparing them.
- Encourage students to think of their ruler, as well as a metre stick or base-ten blocks, when estimating length. Most rulers are 30 cm (or 300 mm) long and serve as good benchmarks. For example, 62 cm can be thought of as the length of about two rulers.
- Ask students to measure objects that do not measure exact centimetres, thus stressing the importance of millimetres when striving for precision of measurement.

SUGGESTED LEARNING TASKS

- Invite students to show, with fingers or arms, the following lengths: 550 mm, 60 cm, 0.25 m. Ask them to describe the length using another unit of measure.
- Ask students to rewrite 3 m using other metric units.
- Invite students to explain what would happen to the numerical value if they measured the length of a long jump in metres and then changed it to centimetres. Would the numerical value become greater or less? Ask them to explain their thinking.

- Share a short paragraph describing the measurements of a variety of classroom items. Have students insert the appropriate unit for each. For example, The table was 1524 ___ long. On it was a pencil that was 10 ___ long.
- Hold a Measurement Scavenger Hunt in the classroom or on the playground. Students should estimate the length of objects first, and then measure for accuracy.
- Ask students to investigate when in our world we would measure with the millimetre unit and why the unit is useful.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- Cuisenaire rods
- measuring tapes
- metre sticks
- rulers
- trundle wheels

MATHEMATICAL LANGUAGE

Teacher	Student
▪ equivalent ▪ length ▪ millimetre, centimetre, metre ▪ referents	▪ equivalent ▪ length ▪ millimetre, centimetre, metre ▪ referents

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 375–377
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 423–425

Notes

SCO M03 Students will be expected to demonstrate an understanding of volume by ▪ selecting and justifying referents for cubic centimetre (cm^3) or cubic metre (m^3) units ▪ estimating volume using referents for cubic centimetre (cm^3) or cubic metre (m^3) ▪ measuring and recording volume (cm^3 or m^3) ▪ constructing rectangular prisms for a given volume [C, CN, ME, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

M03.01 Identify and explain why the cube is the most efficient unit for measuring volume.
M03.02 Provide a referent for a cubic centimetre, and explain the choice.
M03.03 Provide a referent for a cubic metre, and explain the choice.
M03.04 Determine which standard cubic unit is represented by a given referent.
M03.05 Estimate the volume of a given 3-D object using personal referents.
M03.06 Determine the volume of a given 3-D object using manipulatives, and explain the strategy.
M03.07 Construct a rectangular prism for a given volume.
M03.08 Construct more than one rectangular prism for a given volume.

Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
—	**M03** Students will be expected to demonstrate an understanding of volume by ▪ selecting and justifying referents for cubic centimetre (cm^3) or cubic metre (m^3) units ▪ estimating volume using referents for cubic centimetre (cm^3) or cubic metre (m^3) ▪ measuring and recording volume (cm^3 or m^3) ▪ constructing rectangular prisms for a given volume	**M03** Students will be expected to develop and apply a formula for determining the ▪ perimeter of polygons ▪ area of rectangles ▪ volume of right rectangular prisms

Background

Students should explore the idea that one object has more volume than another if it is bigger or takes up more space. The objects used for all of these explorations will be rectangular prisms.

Volume refers to the amount of space that an object takes up. Volume can be measured with cubic centimetres (cm^3) and cubic metres (m^3). Students should have a sense of which attribute, volume or capacity, is the most appropriate to use in any circumstance.

Being able to estimate the volume of various containers and then to measure in the appropriate unit is important as students begin to construct rectangular prisms of various sizes. Students need to have opportunities to develop referents for one cubic centimetre and one cubic metre. For example, students may build a cubic metre with metre sticks as a referent for 1 m^3. A small cube from the base-ten materials may be used as a referent for 1 cm^3. Once students have determined referents, they can begin to think about estimating volume. Students need to consider which unit they might use and about how much the volume is of a shoe box, a cereal box, the classroom, or a refrigerator. Students need to justify their estimations. They can begin to solve problems as a way to explore that the product of the dimensions of a right rectangular prism can be used to find the volume. For example, students can be asked to build rectangular prisms with a volume of 24 cubic units. Providing them with linking cubes allows them to create all possible rectangular prisms and to discover how the dimensions of the rectangular prisms are related to 24. Students could then explore how many cubic centimetres it would take to equal the cubic metre (1 000 000 cm^3 = 1 m^3).

Students should develop a sense of which volume unit is more appropriate to use in various circumstances.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

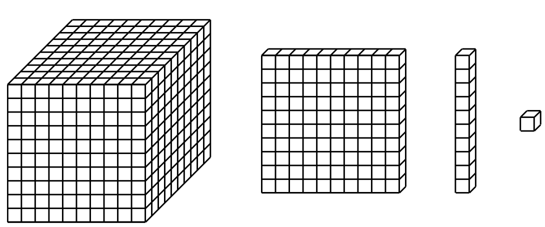
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Invite students to calculate the volume of each size of base-ten blocks.



- Ask students to estimate the volume of the classroom in cubic metres and give an explanation as to how the estimate was determined.
- Ask students to identify a 3-D object that could be measured in cubic centimetres and a 3-D object that would be measured in cubic metres and to explain why those units of measure would be used.
- Tell students that you need to create a box with a volume of 400 cubic centimetres to hold a gift you have purchased. Ask students to identify what that gift might be.
- Invite students to describe how area is different from volume.
- Give students the volume of a rectangular prism and have them construct it using centimetre cubes.
- Ask students to describe the strategy they would use to estimate the volume of certain common rectangular prisms such as lunchboxes, pasta boxes, and tissue boxes.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to discuss a variety of situations where they need to choose the unit of measurement that would be used for each. Have them compare their answers and defend their choices (e.g., the unit of measure to find the volume of the paper clip box, a box of cereal, the volume of a crate that would be used to transport a bicycle, a car, or a dog).
- Provide frequent opportunities for students to construct different rectangular prisms and to discuss the volume of each solid.
- Ask students to find appropriate personal referents for cubic centimetres and cubic metres.
- Use base-ten blocks or linking cubes to build several different structures each with a set volume. Discuss the different dimensions of the rectangular prisms.

SUGGESTED LEARNING TASKS

- Measure the volume of a small rectangular prism by counting the number of centimetre cubes it takes to build a duplicate of it.
- Provide students with a pair of small boxes, one cube, and a ruler. Have students estimate which box has the greater volume and then determine how many cubes would be needed to fill each box. Students should use words, drawings, and numbers to explain their conclusions.
- Invite students to build a cubic metre using metre sticks or other materials. Keep a model to use as a referent for cubic metres.
- Ask students to research the volume of moving trucks. Ask, What is a reasonable estimate for the volume of all the furniture in a school or in a house?
- Give students 24 cubes. Ask them to build a rectangular prism using all 24 cubes so the volume equals 24 cm^3. Invite them to explore how many different rectangular prisms they can construct. Other volumes such as 16, 20, or 36 could also be investigated.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- linking cubes
- measuring tapes
- metre sticks
- rulers
- various models of rectangular prisms

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 3-D object	▪ 3-D object
▪ cube	▪ cube
▪ cubic centimetre, cubic metre	▪ cubic centimetre, cubic metre
▪ constructing rectangular prisms	▪ constructing rectangular prisms
▪ estimating	▪ estimating
▪ referents	▪ referents
▪ volume	▪ volume

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 421–426
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 467–472
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 265–268

Notes

SCO M04 Students will be expected to demonstrate an understanding of capacity by ▪ describing the relationship between millilitre (mL) and litre (L) units ▪ selecting and justifying referents for millilitre (mL) and litre (L) units ▪ estimating capacity using referents for millilitre (mL) and litre (L) ▪ measuring and recording capacity (mL or L) [C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

M04.01 Demonstrate that 1000 millilitres is equivalent to one litre by filling a one-litre container using a combination of smaller containers.
M04.02 Provide a referent for one litre, and explain the choice.
M04.03 Provide a referent for one millilitre, and explain the choice.
M04.04 Determine the capacity unit of a given referent.
M04.05 Estimate the capacity of a given container using personal referents.
M04.06 Determine the capacity of a given container using materials that take the shape of the inside of the container (e.g., a liquid, rice, sand, beads), and explain the strategy.

Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
—	**M04** Students will be expected to demonstrate an understanding of capacity by ▪ describing the relationship between millilitre (mL) and litre (L) units ▪ selecting and justifying referents for millilitre (mL) and litre (L) units ▪ estimating capacity using referents for millilitre (mL) and litre (L) ▪ measuring and recording capacity (mL or L)	**M03** Students will be expected to develop and apply a formula for determining the ▪ perimeter of polygons ▪ area of rectangles ▪ volume of right rectangular prisms

Background

Capacity is the amount a container is capable of holding. Capacity units introduced in Mathematics 5 are millilitres (mL) and litres (L). Capacity units are usually associated with measures of liquid (e.g., milk, juice, medicine, and gasoline).

Since students have had minimal experience with capacity (Mathematics 1, outcome M01), investigation of capacity should begin with direct comparison and non-standard units. Give students containers of different sizes and shapes and ask them to order these containers from largest capacity to smallest capacity by comparing them or using the same small container to fill each. The investigation should next move to the introduction of standard measures. Using a variety of containers can help children see that container shapes can vary but the capacity may remain the same. Students should have the opportunity to compare containers where there is a difference in only one dimension (e.g., the height of the containers is the same, but the bases are different sizes) to explore how this affects the capacity of the containers.

Students should develop personal referents for these units. The use of personal referents helps students establish the relationships between the units. They may think of common items such as a juice box or a milk container or use base-ten blocks (e.g., the small cube in the base-ten blocks has a volume of 1 cm^3 and would hold 1 mL and the large cube has a volume of 1000 cm^3 (10 cm × 10 cm × 10 cm) and would hold 1 L).

It is important for students to estimate capacities and to have a sense of which capacity unit (millilitre or litre) is the more appropriate to use in various circumstances.

In everyday usage, volume and capacity are often confused.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

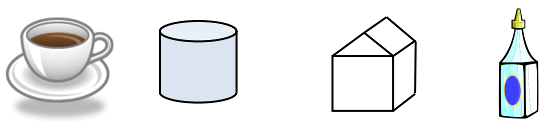
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students which capacity unit they would use to measure the capacity of the following and to explain their choices.
  - swimming pool
  - coffee mug
  - bottle
  - bathtub
  - juice glass
- Tell students that a jug holds 1.5 L. Ask students to determine if the jug is large enough to make a jug of orange juice, if you must mix a 355 mL can of frozen orange juice concentrate with three full cans of water. Ask them to explain their thinking.

- Ask students to explain how they could use a 1 L milk carton to estimate 750 mL of water.
- Provide students with three or four different containers and content to measure (liquid, sand, rice). Invite students to determine the capacities of the containers and explain their strategy.
- Invite students to describe their personal referent for millilitres and litres and to explain why each referent was selected.
- Ask students to describe the strategy they would use to estimate the capacity of certain common containers such as water bottles, bathtubs, or various milk containers.
- Provide a number of small containers that have labels showing the capacity of each in millilitres. Ask students to determine combinations of containers that would fill a one-litre container.



250 mL 150 mL 500 mL 100 mL

Follow-up on Assessment

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Ask students to discuss a variety of situations where they need to choose the unit of measurement that would be used for each. Have students compare their answers and defend their choices. For example, the unit of measure to find the capacity of cough syrup bottles, water bottles, juice boxes, yogurt containers, bathtubs, and gas tanks.

- Invite groups of students to investigate the capacities of various beverage containers to determine the capacity of the container that is found most often.
- Provide ample opportunity for students to measure the capacity of different shaped and sized containers. Have students predict which unit of measure will be used.
- Ask students to find appropriate personal referents for litres and millilitres. They might look for containers at home and bring in empty samples.

Possible Referents	
Litres	**Millilitres**
water bottle	eye dropper
milk jug	spoon
liquid detergent bottle	small yogurt container

SUGGESTED LEARNING TASKS

- Ask students to measure the capacity of several different containers and record the most common capacities.
- Provide students with a pair of containers and ask them to predict which has the largest capacity (which holds more). Have them verify their predictions.
- Invite students to compare several cereal bowls to see how much a typical bowl can hold.
- Invite students to estimate the number of beans needed to fill a litre container and then check their estimation.
- Ask students to suggest containers for fixed capacities (e.g., What kind of containers would hold 500 mL, 1 L, or 250 mL?)
- Provide students with a variety of containers (scoops, cups, and spoons) and ask them to estimate how many of one container it would take to fill another. To determine if their estimation was correct, students would fill the large container from the smaller to check.
- Invite students to order various containers based on their capacity. Students should provide a referent for each of the different containers (e.g., 250 mL is about the same as a small milk container). In their journals, students should illustrate and explain how they know their ordering is correct.
- Tell students that Jim has to make a recipe in which he has to use 2 L of orange juice, and he only has a 500-mL container to measure the juice. Ask students to explain using numbers, pictures, and words how Jim could use the 500-mL container to measure 2 L of orange juice.
- Ask students to draw three smaller containers whose capacity, when added, would equal 1 L. Ask them to explain their choices.
- Ask students to choose from a series of containers (5 mL, 75 mL, 200 mL, etc.), a combination that will create a total capacity of 1 litre.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- measuring cup and spoons (metric)
- various containers

MATHEMATICAL LANGUAGE

Teacher	Student
▪ capacity ▪ litre ▪ millilitre	▪ capacity ▪ litre ▪ millilitre

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 418–421
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 466–467
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 265–268

Notes

Geometry (G)

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

SCO G01 Students will be expected to describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are parallel, intersecting, perpendicular, vertical, and horizontal. [C, CN, R, T V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G01.01 Identify parallel, intersecting, perpendicular, vertical, and horizontal edges and faces on 3-D objects.

G01.02 Identify parallel, intersecting, perpendicular, vertical, and horizontal sides on 2-D shapes.

G01.03 Provide examples from the environment that show parallel, intersecting, perpendicular, vertical, and horizontal line segments.

G01.04 Find examples of edges, faces, and sides that are parallel, intersecting, perpendicular, vertical, and horizontal in print and electronic media, such as newspapers, magazines, and the Internet.

G01.05 Draw 2-D shapes that have sides that are parallel, intersecting, perpendicular, vertical, or horizontal.

G01.06 Build 3-D objects that have edges and faces that are parallel, intersecting, perpendicular, vertical, or horizontal.

G01.07 Describe the faces and edges of a given 3-D object using terms such as **parallel**, **intersecting**, **perpendicular**, **vertical**, or **horizontal**.

G01.08 Describe the sides of a given 2-D shape using terms such as **parallel**, **intersecting**, **perpendicular**, **vertical**, or **horizontal**.

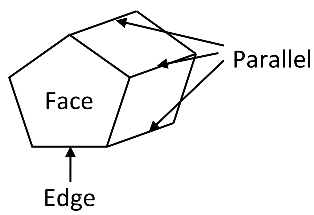
Scope and Sequence

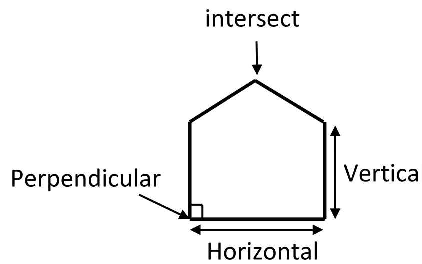
Mathematics 4	**Mathematics 5**	**Mathematics 6**
G01 Students will be expected to describe and construct rectangular and triangular prisms. **Note:** 2-D shapes are addressed in Mathematics 3.	**G01** Students will be expected to describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are parallel, intersecting, perpendicular, vertical, and horizontal.	**G01** Students will be expected to construct and compare triangles, including scalene, isosceles, equilateral, right, obtuse, or acute in different orientations.

Background

There is a gradual progression from identifying and describing 2-D shapes and 3-D objects in students' own words to identifying and describing them in the formal language of geometry. It is important that students become familiar with the vocabulary associated with describing the attributes of 2-D shapes and 3-D objects such as **parallel**, **intersecting**, **perpendicular**, **vertical**, and **horizontal**.

Lines in the same plane can be parallel or they can intersect. Parallel lines never intersect since they remain a constant distance apart. Intersecting lines meet at a single point. Perpendicular lines are intersecting lines that form right angles. Lines can also be vertical or horizontal. Vertical lines are up and down and are perpendicular to the horizon. Horizontal lines are parallel to the horizon or are from left to right.





Students will also be expected to compare and describe 2-D shapes by relating their attributes and will also be expected to compare and describe 3-D objects in the same way. When given a set of attributes, students should be able to construct or draw the 2-D shape or 3-D object that corresponds to the description. A good source for exploring the properties of 2-D shapes and 3-D objects is found at http://illuminations.nctm.org/ActivityDetail.aspx?ID=70 (National Council of Teachers of Mathematics 2014).

In order to make generalizations about the properties of shapes, students should be guided to investigate 2-D shapes and 3-D objects by working directly with models. Students should be working towards an understanding of concepts and skills related to geometry rather than being given definitions and rules to memorize. They should be exposed to a variety of experiences, which allow them to explore shapes in order to discover properties of shapes independently. These explorations will support a greater level of comfort and confidence in using mathematics to make sense of real-life situations. Exploration with models could include technologies such as Geometer's Sketchpad and web-based tools known as virtual manipulatives. These tools help students create a conceptual understanding of geometry. While the language of geometry is important, the teaching of mathematically correct geometric language should be done in the context of physical models rather than as definitions.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to work together to sort a collection of 3-D objects into two groups: rectangular prisms and triangular prisms. Have students identify and explain the attributes of the objects that made them alike. Ask students to identify and explain the attributes of the objects that made them different.
- Ask students to identify and explain the attributes of a cube that makes it a rectangular prism.
- Have students identify and explain the kind of prism that could be built from a rectangular base.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Using a variety of geometric solids, have students identify parallel, intersecting, and perpendicular edges.
- Provide students with several different 2-D shapes and have them sort and justify their sorting scheme. Observe whether students use correct geometric terminology in their descriptions.
- Provide students with several different 3-D objects and have them sort them and justify their sorting scheme. Observe whether students use correct geometric terminology in their descriptions.
- Ask students to draw 2-D shapes and 3-D objects that satisfy a given set of attributes. For example, draw a parallelogram with both parallel and perpendicular sides (rectangle).
- Complete a Venn or Carroll diagram focusing on the attributes of 2-D shapes and 3-D objects as shown below.

Attributes	2-D object	3-D shape
Has parallel sides, edges, or faces.		
Does not have any parallel sides, edges, or faces.		

Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Provide opportunities for students to manipulate 2-D shapes and 3-D objects and become familiar with the vocabulary associated with describing the attributes of 2-D shapes and 3-D objects, such as parallel, intersecting, perpendicular, vertical, and horizontal.
- Encourage students to use the formal language of geometry as they describe the attributes of classroom or real-life 2-D shapes and 3-D objects (e.g., the opposite walls in the classroom are parallel).
- Ask students to trace pattern blocks, examine the sides of the shapes they have traced, categorize the sides as parallel, intersecting, perpendicular, vertical, or horizontal.
- Invite students to stack pattern blocks to build prisms. (Pattern blocks will form triangular prisms, rectangular prisms, trapezoidal prisms, rhomboidal prisms, and hexagonal prisms.) Ask students to examine the 3-D objects they create and to categorize the faces or edges of each as parallel, intersecting, perpendicular, vertical, and horizontal.
- Go on a walk to look at 2-D shapes and 3-D objects in the environment. Have students discuss the attributes of shapes and objects in their environment using geometric language.

Suggested Learning Tasks

- Ask students to draw and construct on a geo-board 2-D shapes with specific attributes (e.g., construct a shape with a pair of parallel sides).
- Invite students to construct 3-D objects with toothpicks and marshmallows and have them describe their attributes.
- Ask students to sort 2-D shapes and 3-D objects according to their attributes and justify their sorting scheme.
- Compare and describe the faces and edges of two prisms or two pyramids with different bases (e.g., triangular prisms and rectangular prisms).
- Have students search through newspapers, magazines, etc. to find examples of vertical and horizontal lines.

- Using a variety of geometric solids, ask students to identify parallel, intersecting, and perpendicular edges.
- Invite students to construct 3-D objects using wooden craft sticks and glue and ask them to paint them different colours according to any given properties.

Suggested Models and Manipulatives

- geo-boards
- geo-strips
- geometric solids
- grid paper
- pattern blocks
- toothpicks and marshmallows

Mathematical Language

Teacher	Student
▪ 2-D shapes and 3-D objects ▪ edges, faces, sides ▪ parallel, intersecting, perpendicular, vertical, horizontal, perpendicular, perpendicular bisector ▪ Venn and Carroll diagrams	▪ 2-D shapes and 3-D objects ▪ edges, faces, sides ▪ parallel, intersecting, perpendicular, vertical, horizontal, perpendicular, perpendicular bisector ▪ Venn and Carroll diagrams

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 292–295
- Mak*ing Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 348–351
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 222–224, 226
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 193–195

Internet

- Illuminations: Resource for Teaching Math, “Geometric Solids” (National Council of Teachers of Mathematics 2014) http://illuminations.nctm.org/ActivityDetail.aspx?ID=70

Notes

SCO G02 Students will be expected to name, identify, and sort quadrilaterals, including rectangles, squares, trapezoids, parallelograms, and rhombi, according to their attributes. [C, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G02.01 Identify and describe the characteristics of a pre-sorted set of quadrilaterals.
G02.02 Sort a given set of quadrilaterals, and explain the sorting rule.
G02.03 Sort a given set of quadrilaterals according to the lengths of the sides.
G02.04 Sort a given set of quadrilaterals according to whether or not opposite sides are parallel.
G02.05 Sort a set of quadrilaterals based on properties such as diagonals are congruent, diagonals bisect each other, and opposite angles are equal.
G02.06 Name and classify quadrilaterals according to their attributes.

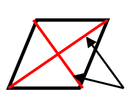
Scope and Sequence

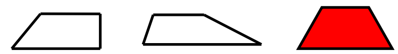
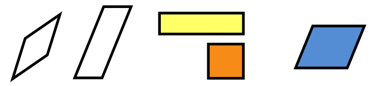
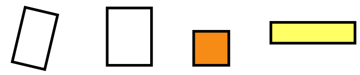
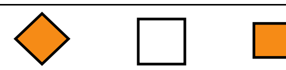
Mathematics 4	Mathematics 5	Mathematics 6
G03 Students will be expected to demonstrate an understanding of line symmetry by ▪ identifying symmetrical 2-D shapes ▪ creating symmetrical 2-D shapes ▪ drawing one or more lines of symmetry in a 2-D shape	**G02** Students will be expected to name, identify, and sort quadrilaterals, including rectangles, squares, trapezoids, parallelograms, and rhombi, according to their attributes.	**G02** Students will be expected to describe and compare the sides and angles of regular and irregular polygons.

Background

Quadrilaterals are polygons. Polygons are closed, two-dimensional figures. All quadrilaterals have four straight sides and four angles. “Although rectangles are the most common quadrilateral that you see in everyday life, students will soon discover that there are many classes of quadrilaterals.” (Small 2008b, 295) Students will be exploring the attributes of various quadrilaterals such as **trapezoids**, **parallelograms**, **rectangles**, **rhombi**, and **squares**. They will compare the similarities and differences and sort them according to their attributes.

Students need to sort quadrilaterals based upon their attributes. Common attributes may be lengths of sides, pairs of opposite parallel sides, lines of symmetry, and diagonals. All quadrilaterals have two diagonals (line segments that joins two non-adjacent vertices). It is important for students to recognize that some quadrilaterals can be classified in more than one category (e.g., a square is also a rectangle and a parallelogram).



Quadrilateral	Attributes	Examples
Trapezoid	One pair of parallel sides. (**Note:** An isosceles trapezoid has a pair of opposite sides that are congruent [shaded example].)	
Parallelogram	Two pairs of parallel sides. Opposite sides are congruent. Opposite angles are congruent.	
Rhombus	A parallelogram with all sides congruent and opposite angles congruent.	
Rectangle	A parallelogram with four right angles. Two pairs of parallel sides. Opposite sides are congruent. Opposite angles are congruent.	
Square	A parallelogram with four right angles and all sides congruent.	

Students need to consider statements to determine whether or not they are true. For example,

- All squares are rectangles.
- All squares are rhombi.
- All rectangles are squares.

Students must be able to justify their answers.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

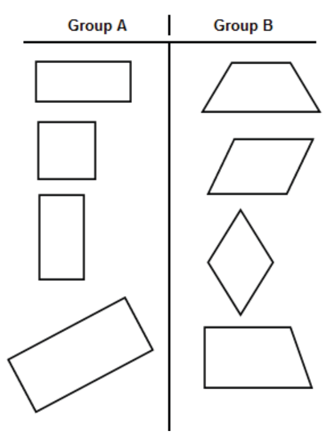
Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to name different 2-D shapes, to state how many lines of symmetry each has, and to show where the lines of symmetry are.
- Provide students with a set of 2-D shapes. Ask them to sort the shapes and to explain their sorting rule.
- Provide students with a set of 3-D objects. Ask them to sort the objects and to explain their sorting rule.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with several different quadrilaterals to sort and have them justify their sorting rule. Invite them to sort the shapes a different way and describe their sorting rule.
- Ask students to draw quadrilaterals that satisfy a given set of attributes. Be sure to include in the drawings the lengths of the sides and whether or not the opposite sides are parallel. Once they have the quadrilateral drawn, they should be able to identify the shape. For example,
  - a 2-D shape with four straight sides of equal length and four right angles
  - a 2-D shape with four straight sides and four right-angles (One pair of sides is longer than the other.)
  - a 2-D shape with four straight sides (One pair of sides is parallel with one side longer than the other.)
- Provide students with a pre-sorted set of quadrilaterals and ask them to identify the sorting rule.



- Provide students with a pre-sorted set of quadrilaterals. Show them another quadrilateral and ask, Where should I place this quadrilateral? Ask students to explain their thinking.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

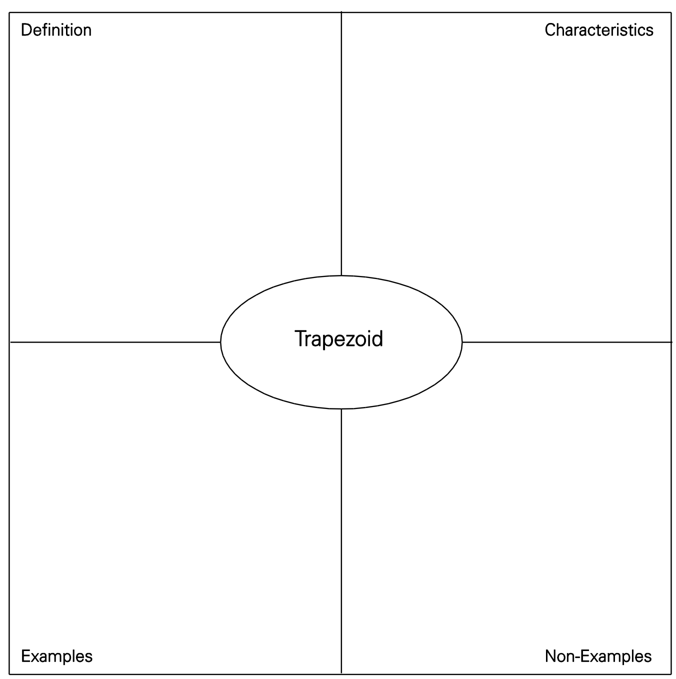
Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use models, drawings, and real-life examples of quadrilaterals, to identify and describe the characteristics of each and classify them. Have students explain their classification system.
- Provide the students with a template for the Frayer Model and have them fill in the sections individually to demonstrate their understanding of the properties of a particular quadrilateral, such as a trapezoid.



SUGGESTED LEARNING TASKS

- Ask students to go on a Quadrilateral Scavenger Hunt around the school. Students could take pictures of the shapes they find and then use the picture to sort the quadrilaterals with similar attributes. Other students could be asked to view the sorted shapes and to explain the sorting rule that was used.
- Using a collection of quadrilaterals, invite students to describe the shapes using correct mathematical language, such as "opposite sides equal," "all sides equal," "no sides equal," or "two pairs of parallel sides," "one pair of parallel sides," or "no sides parallel."
- Provide students with a list of attributes and have them identify a quadrilateral that has the specified set of attributes. Have students share and compare with the class.
- Prepare a Guess What Quadrilateral I Am? game with clues about their attributes. The questions must have a "yes" or "no" answer (e.g., Are opposite sides the same length?).

SUGGESTED MODELS AND MANIPULATIVES

- 2-D shapes
- geo-boards
- geo-strips
- geometric solids
- grid paper
- pattern blocks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ bisect	▪ bisect
▪ characteristics	▪ characteristics
▪ congruent	▪ congruent
▪ diagonals	▪ diagonals
▪ Frayer Model	
▪ quadrilaterals	▪ quadrilaterals
▪ rectangles, squares, trapezoids, parallelograms, and rhombi	▪ rectangles, squares, trapezoids, parallelograms, and rhombi

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 295–296, 518–524
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 351–352, 562–566
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 221–224, 226
- *Teaching Student-Centered Mathematics, Grades 5–8*, (Van de Walle and Lovin 2006), p. 198

Notes

SCO G03 Students will be expected to perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image. [C, CN, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G03.01 Translate a given 2-D shape horizontally, vertically, or diagonally, draw the image, and describe the position and orientation of the image.

G03.02 Rotate a given 2-D shape about a vertex, draw the image, and describe the position and orientation of the image.

G03.03 Reflect a given 2-D shape in a line of reflection, draw the image, and describe the position and orientation of the image.

G03.04 Perform a transformation of a given 2-D shape by following instructions.

G03.05 Draw a 2-D shape, translate the shape, and record the translation by describing the direction and magnitude of the movement.

G03.06 Draw a 2-D shape, rotate the shape about a vertex, and describe the direction of the turn (clockwise or counter-clockwise) and the fraction of the turn (limited to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or full turn).

G03.07 Draw a 2-D shape, reflect the shape, and identify the line of reflection and the distance of the image from the line of reflection.

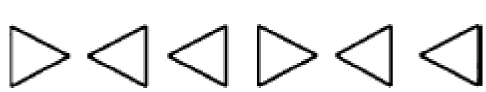
G03.08 Predict the result of a single transformation of a 2-D shape and verify the prediction.

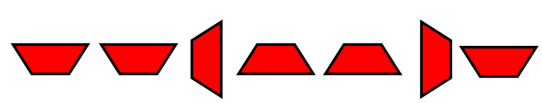
Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
—	**G03** Students will be expected to perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image.	**G03** Students will be expected to perform a combination of translation(s), rotation(s), and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image.

Background

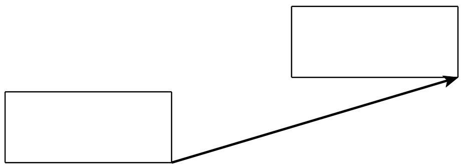
Mathematics 5 is the first time students are introduced to transformations. Students should be encouraged to observe, describe, and create patterns using translations, reflections, and rotations. Pattern blocks are an ideal concrete material for students to manipulate when creating patterns.



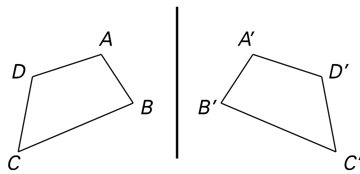


There are three types of transformations that will be explored in this grade: translation (slide), reflection (flip), and rotation (turn).

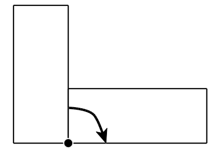
Example of a translation (slide)



Example of a reflection (flip)



Example of a rotation (turn)



Introducing transformations to students, the teacher should begin with informal language (slide, flip, and turn) making the connection to mathematical terms (**translation**, **reflection**, and **rotation**). Encouraging students to think in terms of sliding, flipping, and turning shapes is a very useful strategy to help with their visualization in geometry since much of students' geometric development depends on understanding how shapes do and do not change when they are transformed in different ways. Students could begin by using concrete materials such as pattern blocks and geo-boards to demonstrate translations, reflections, and rotations. Once they have concretely demonstrated understanding of performing the transformations, they can use square dot paper or grid paper to draw the transformations. Various manipulatives can be used to investigate transformations on grid paper, such as cardboard cut-outs and geometry sets. Students should identify how the pre-image (original) shape and the image are related. This topic is also well suited to the use of software such as Geometer's Sketchpad (Key Curriculum 2013), the draw program in Word, and the Microsoft Paint program.

Introducing translations (slide), students should develop intuitive notions that a slide moves a shape up or down, right or left, without rotating (turning) it or reflecting (flipping) it. To describe slides at the beginning, students might use such descriptors as "to the right and up" but should learn to describe the translation accurately as right two, up four. By drawing connecting lines between vertices in the pre-image to corresponding vertices in the image, students can see that these lines are all congruent and that each point has been moved the same distance. Similarly, these lines are all parallel and thus each point has been moved in the same direction. Also, students should observe that the image and pre-image are congruent.

Rotations (turn) are the most perceptually challenging of the transformations. Students need many first-hand experiences making rotations and examining the results before they will be able to identify rotations. With both drawing and identifying rotational images, the emphasis should be on rotations of one-quarter, one-half, three-quarters, and full turns, both clockwise and counter-clockwise. Students should explore these turns of shapes with their vertices as centres. Students should observe congruence between the image and pre-image.

A reflection (flip) does change the orientation of a shape; the effect is one of reversing a shape (i.e., right becomes left or up becomes down). To describe reflections (flips), students should be able to use language such as, "reflected up" or "reflected to the left." Students should draw or trace shapes, and using a Mira, draw mirror lines, and the reflected images. Students should then compare the shapes and their reflected images using tracing paper or by folding over and looking through the paper at a light source. They should conclude that the two shapes are congruent. Students should label the original shape with *A*, *B*, *C*, *D*, ...; the corresponding vertices of the reflected image with prime notation, *A', B', C', D'*. They should name both shapes clockwise starting at *A* and *A'*. Students should conclude that the original shape and its image are of opposite orientation. Students should also join corresponding vertices with line segments and examine the angles made by the mirror lines with these segments. They should conclude that the mirror lines are perpendicular to all segments joining corresponding image points. Students should also measure the distance from corresponding vertices to the mirror line. They should conclude that corresponding points are equidistant from the mirror lines. In short, the mirror line is the perpendicular-bisector of all segments joining corresponding points.

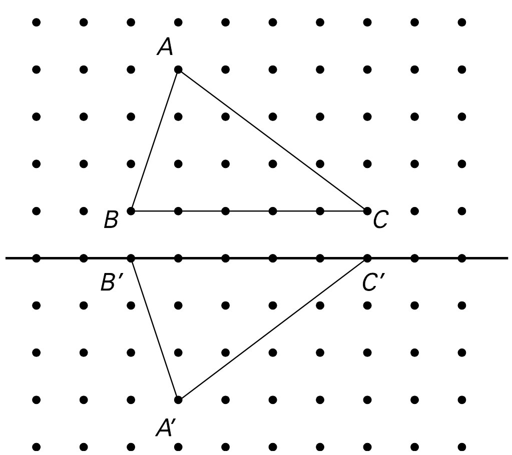
Once students have created images and pre-images, they should be able to talk about how they are the same and how they are different. In discussing the transformations, students should consider if the image

- has side lengths congruent to those of the pre-image
- has angle measures congruent to those of the pre-image
- is congruent to the pre-image
- has the same orientation as the pre-image
- have corresponding sides parallel

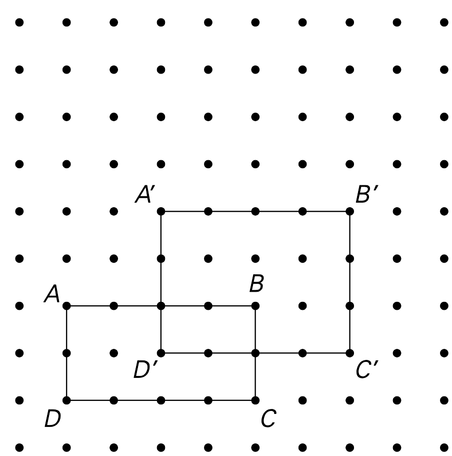
Eventually, notation for pre-image and image of transformations should be introduced and developed in this grade. One method students can use is “prime notation,” another could be renaming the image points. While eventually we will want students to use prime notation proficiently, what is more important is that they can distinguish the images from each other using mathematical notation knowing that the pre-image is the original figure and the image is the result of the transformation.

Students should be able to predict where an image will be before performing the transformation. This is a similar process to estimating the result of a calculation before performing the calculation. Teachers should encourage students to make such predictions as a way to develop spatial reasoning.

Students should also be able to explain whether a transformation is or is not the result of a given translation. For example, students should be able to explain why the diagrams below are non-examples of transformations.

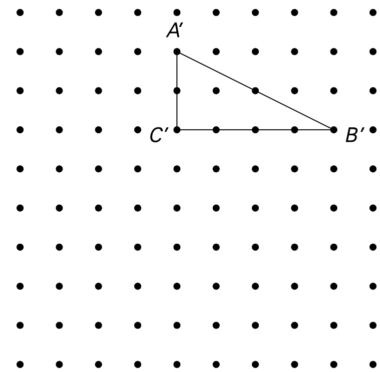


This does not show a reflection in the given line because the triangles are not the same distance from the line.



This does not show a slide of right two, up two, because the rectangle at the top is bigger than the rectangle on the bottom.

Students should explore identifying the position of the pre-image (what we started with) when given the image under a given transformation. This will help students to understand that there are inverse transformations that can map the image back to its pre-image. For example, if the image shown below was formed on the geo-board by sliding right two and up three, where was the original triangle?



Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide a 2-D shape and have students show a rotation, reflection, or translation of that shape on grid paper.
- Invite students to draw a shape, translate it, and then describe and explain the direction and the magnitude of the translation.
- Ask students to use their hands to demonstrate the three different transformations.
- Invite students to explain the differences and similarities among the three different transformations.
- Invite students to explain, using words and pictures, if a translation can ever look like a reflection.
- Provide students with pre-images and images of shapes that have undergone a transformation. Ask students to select one of the shapes, identify the transformation that it has undergone, and to explain their thinking.

Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Provide students with many opportunities to concretely translate a given 2-D shape using a geo-board and grid paper. Ensure that the direction and the magnitude of the translation is described.
- Provide students with many opportunities to concretely reflect a given 2-D shape using a Mira and grid paper. Ensure that the line of reflection is identified and that the distance from the line of reflection is described.
- Provide students with many opportunities to concretely rotate a given 2-D shape using a geo-board and grid paper. Ensure that a point of rotation is identified and that the fraction and direction of the turn is described.
- Explore this concept in other curricular areas such as visual arts and physical education. Invite students to create their own wallpaper patterns using different transformations or to act out a transformation in the gym.
- Ask students to discuss their predictions prior to performing a given transformation to a shape.
- Use technology-based activities from NCTM Illuminations (2014), which can be found at http://illuminations.nctm.org.

SUGGESTED LEARNING TASKS

- Ask students to draw a shape and practise the different types of transformations using their shape. The transformations could be drawn on grid paper.
- Invite students to draw a shape, perform a transformation of their choice, draw the transformation on grid paper, and have a partner describe the transformation that was performed.
- Ask students to perform a translation given the direction and magnitude of the movement.
- Ask students to perform a reflection given the line of reflection and the distance from the line of reflection.
- Invite students to perform a rotation given the direction of the turn (clockwise or counter-clockwise), the fraction of the turn (limited to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or full turn), and the turn centre (point of rotation). Turn centres should be one of the vertices of the shape.
- Ask students to create a shape on the geo-board, perform a transformation of their choice, and describe the transformation that was performed.

SUGGESTED MODELS AND MANIPULATIVES

- geo-boards
- grid paper
- Miras
- Geometer's Sketchpad (Key Curriculum 2013)
- Microsoft Paint
- pattern blocks
- tracing paper

MATHEMATICAL LANGUAGE

Teacher	Student
▪ $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or full turn ▪ direction and fraction of turn ▪ direction and magnitude ▪ horizontally, vertically, diagonally ▪ position and orientation ▪ pre-image, image ▪ predict ▪ prime notation ▪ transformation ▪ translation, rotation, reflection ▪ vertices	▪ $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or full turn ▪ direction and fraction of turn ▪ direction and magnitude ▪ horizontally, vertically, diagonally ▪ position and orientation ▪ pre-image, image ▪ predict ▪ prime notation ▪ transformation ▪ translation, rotation, reflection ▪ vertices

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 342–349, 355–356
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 392–399, 405–406
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 233–234
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 209–210

Internet

- *Illuminations: Resources for Teaching Math* (National Council of Teachers of Mathematics 2014) http://illuminations.nctm.org

Software

- Geometer's Sketchpad (Key Curriculum 2013)

Notes

SCO G04 Students will be expected to identify and describe a single transformation, including a translation, rotation, and reflection of 2-D shapes. [C, T, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

G04.01 Provide an example of a translation, rotation, and reflection.
G04.02 Identify a given single transformation as a translation, rotation, or reflection.
G04.03 Describe a given rotation about a point of rotation by the direction of the turn (clockwise or counter-clockwise).
G04.04 Describe a given reflection by identifying the line of reflection and the distance of the image from the line of reflection.
G04.05 Describe a given translation by identifying the direction and magnitude of the movement.
G04.06 Identify transformations found in everyday pictures, art, or the environment.

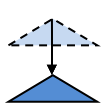
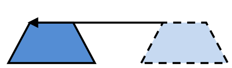
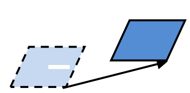
Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
G03 Students will be expected to demonstrate an understanding of line symmetry by ▪ identifying symmetrical 2-D shapes ▪ creating symmetrical 2-D shapes ▪ drawing one or more lines of symmetry in a 2-D shape	**G04** Students will be expected to identify and describe a single transformation, including a translation, rotation, and reflection of 2-D shapes.	**G04** Students will be expected to perform a combination of successive transformations of 2-D shapes to create a design and identify and describe the transformations.

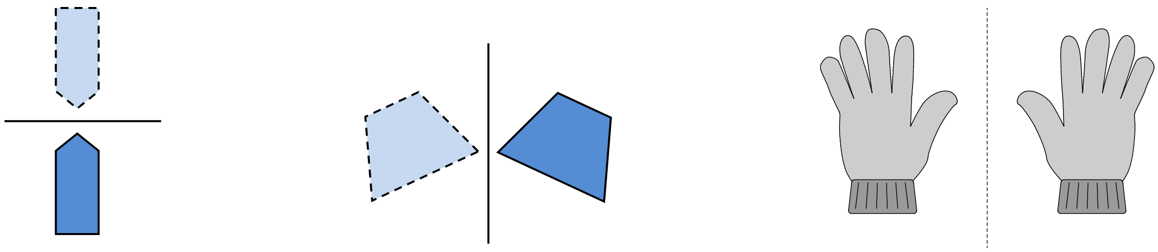
Background

There are three transformations that change the location of an object in a plane. The three types of transformations are: **translations**, **reflections**, and **rotations**. These transformations result in images that are congruent to the original object. Students are expected to identify these three types of transformations.

Translations move a shape left, right, up, down, or diagonally without changing its orientation in any way. A real-life example of a translation would be a piece moving on a chessboard.

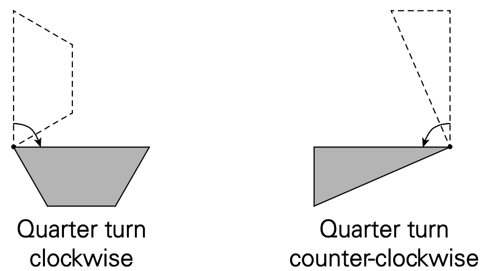




Reflections can be thought of as the result of picking up a shape and flipping it over. The reflection image is the mirror image of the original shape. A real-life example of a reflection could be a pair of gloves beside each other.



The line of reflection creates symmetry between the pre-image and image, whereas a line of symmetry typically refers to symmetry within a given object.

Rotations move a shape about a **turn centre**. When students first start working with rotations, they identify the amount of rotation as fractions of a circle. In Mathematics 5, rotations will be limited to a **quarter turn**, a **half turn**, a **three-quarter turn**, and a **full turn**. Students are also expected to identify the direction of the rotation; **clockwise** and **counter-clockwise**. A real-life example of a rotation would be the movement of the hands on a clock.



Students could look for the use of transformations by searching the Internet. They could examine the transformations used by artists of various cultures including First Nations and African Nova Scotians, wallpaper designers, and quilt makers.

Please refer to outcome G03 for additional information.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

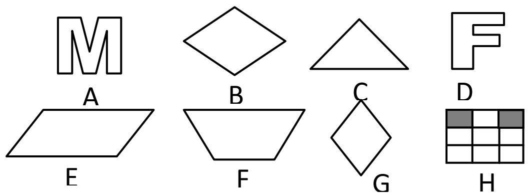
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Place the following labelled 2-D shapes before the student.



Ask students to circle all the symmetrical shapes. Then invite students to draw all the lines of symmetry on the symmetrical shapes. Finally, ask students to sort the shapes by the number of lines of symmetry in each shape: no lines of symmetry, one line of symmetry, and more than one line of symmetry.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with diagrams of different transformations and ask them to label each diagram with the type of transformation the diagram showed.
- Provide diagrams of rotations and ask, Which picture shows a quarter turn? Half turn? Three-quarter turn? Invite students to identify the turn centre of the rotation.
- Explain using words and pictures how you know if a figure and its image show a translation, reflection, or rotation.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with samples of wallpaper and have them explore various types of transformations.

SUGGESTED LEARNING TASKS

- Ask students to describe the direction as well as the magnitude of a given translation.
- Ask students to determine which transformation was performed on a given shape.
- Invite students to respond in their journal to the following prompts:
  - Explain using words and pictures if a translation can ever look like a reflection.
  - Explain using words and pictures how you know if a figure and its image show a reflection, translation, or rotation.

SUGGESTED MODELS AND MANIPULATIVES

- geo-boards
- grid paper
- Miras
- pattern blocks
- tracing paper

MATHEMATICAL LANGUAGE

Teacher	Student
▪ $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or full turn ▪ direction and fraction of turn ▪ direction and magnitude ▪ horizontally, vertically, diagonally ▪ point of rotation ▪ position and orientation ▪ pre-image, image ▪ predict ▪ prime notation ▪ transformation ▪ translation, rotation, reflection ▪ turn centre ▪ vertices	▪ $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or full turn ▪ direction and fraction of turn ▪ direction and magnitude ▪ horizontally, vertically, diagonally ▪ point of rotation ▪ position and orientation ▪ pre-image, image ▪ predict ▪ prime notation ▪ transformation ▪ translation, rotation, reflection ▪ turn centre ▪ vertices

Resources/Notes

Print

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- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 392–399, 405–406
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 233–234
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 209–210

Notes

SCO G05 Students will be expected to identify right angles. [ME, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G05.01 Provide examples of right angles in the environment.
G05.02 Sketch right angles without the use of a protractor.
G05.03 Label a right angle, using a symbol.
G05.04 Identify angles greater than or less than a right angle.

Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
—	**G05** Students will be expected to identify right angles.	**G02** Students will be expected to describe and compare the sides and angles of regular and irregular polygons.

Background

Examples of right angles should be presented in a variety of positions so students do not associate the word “right” in any way with how the angle points. Discuss the everyday meaning of the word, “right,” including “right” meaning correct, and “right” as a direction. Classroom objects provide many examples of right angles, and these should be explored, drawn, and discussed. Logic blocks provide examples of shapes that have right angles and shapes that do not. Since most notebook pages are rectangular, the corners of these pages provide a referent for right angles.

Presenting students with pictures and asking them to find as many right angles as possible assists them in viewing different positions for right angles. As well, students should identify right angles in various 2-D shapes and 3-D objects and label these angles appropriately. They can be further challenged by asking them to find an angle that is less than or greater than a right angle. They should also draw or create right angles in many different positions in space.

Students are not measuring angles in degrees at this stage; they are comparing angles by sight. Having students identify right angles (90° angles) provides them with a benchmark for estimating angle size. They should describe angles as less than or more than a right angle. Students should examine a variety of polygons, including familiar triangles and quadrilaterals, to identify angles that are right, greater than right, and less than right.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Invite students to make shapes on geo-boards that meet different criteria. For example, make shapes that have two right angles; make shapes that have one right angle; make a shape that has one angle that is greater than a right angle; make shapes that have all their angles less than right angles.
- Provide students with pictures of 12 different angles in different positions on the paper. Ask them to sort the angles into three sets: right angles, more than right angles, and less than right angles.
- Ask students to sketch a right angle and explain the strategy they used to sketch the angle.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

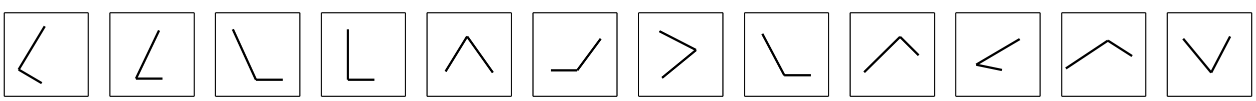
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Connect work on this outcome with the exploration of quadrilaterals (G02).
- Use models, drawings, and real-life examples of quadrilaterals, to identify and describe the characteristics of each and classify them. Have students explain their classification system.

SUGGESTED LEARNING TASKS

- Provide students with drawings of right angles, greater than right angles, and less than right angles in different positions. Invite students to label each angle and to explain their thinking.
- Ask students to draw a right angle in two different positions on their paper.
- Tell students that Jeri went on a trip with her parents. To amuse herself, she sketched the ways roads met at intersections. The following are some of her drawings. Ask students to identify how many of the angles formed by the roads were right angles, how many were greater than right angles, and how many were less than right angles.



- Find angles in the environment, in the classroom, or in the school that would be right, less than right, and more then right.
- Provide students with small rectangular pieces of paper or file cards. Discuss right angles in relation to the corners of these items. Ask pairs of students to gather a set of six pattern blocks, and using the corner of the file card, find shapes that have right angles, angles less than right angles, and angles greater than right angles.
- Ask students to examine various members of the quadrilateral family of shapes to identify right angles, angles less than right angles, and angles greater than right angles.
- Invite students to arrange toothpicks to make right angles and a variety of angles less than or greater than right angles.
- Give students 12 cards with examples of right angles, greater than right angles, and less than right angles on them. Ask them to sort the angles into three groups by the nature of their angles and share how they were sorted.
- Invite students to look for examples in the world of each type; examine familiar material in the classroom (e.g., pattern blocks, tangrams).
- Ask students to choose straws of different lengths or geo-strips to make examples of each type of angle.

SUGGESTED MODELS AND MANIPULATIVES

- 2-D shapes and 3-D objects having right angles
- geo-boards
- index cards
- pictures of angles, in different positions, including right angles, greater than right angles, and less than right angles

MATHEMATICAL LANGUAGE

Teacher	Student
▪ angle ▪ right angle ▪ position in space	▪ angle ▪ right angle ▪ position in space

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), p. 461
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), p. 507
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 207, 213, 216
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), p. 191

Notes

Statistics and Probability (SP)

GCO: Students will be expected to collect, display, and analyze data to solve problems.

GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

SCO SP01 Students will be expected to differentiate between first-hand and second-hand data. [C, R, T, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

SP01.01 Explain the difference between first-hand and second-hand data.
SP01.02 Formulate a question that can best be answered using first-hand data and explain why.
SP01.03 Formulate a question that can best be answered using second-hand data and explain why.
SP01.04 Find examples of second-hand data in print and electronic media, such as newspapers, magazines, and the Internet.

Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
—	**SP01** Students will be expected to differentiate between first-hand and second-hand data.	**SP02** Students will be expected to select, justify, and use appropriate methods of collecting data, including questionnaires, experiments, databases, and electronic media.

Background

Students are familiar with collecting and organizing data from previous grades. In Mathematics 5, students will learn about **first-hand data** that they collect themselves and **second-hand data** that other people have collected. The focus will be on comparing the collection methods and communicating results.

First-hand data is data that is collected by the researcher (in this case the students) and is best used when they are looking for answers to questions about people, places, or objects found in their everyday lives. First-hand data is required when this information is not readily available from existing respectable sources. Collecting first-hand data can be done using a variety of methods such as interviews, surveys, experiments, and observations. Students will need to determine what data they want to collect, gather the data, and then analyze it using **reasoning** to draw conclusions.

Students have had many experiences collecting data in their early years in school; most will have found the favourite foods of other students, lengths of names, or what pets their classmates have. This type of data collection should continue. Students should be aware that there are many ways to collect data and that these various methods may provide slightly different results (e.g., Students could consider the difference in data collected about favourite foods if they simply ask each classmate to list their favourite, as opposed to offering a choice of three foods and asking students which of the three they prefer. They might also consider the difference in results concerning favourite foods if they collect data right before lunch rather than at another time of the day.)

Second-hand data is data that has been collected by someone else. It can be found in print and electronic media. Students will need to create appropriate questions that can be answered using second-hand data, and then use that data to communicate different conclusions. Students should understand that when data is needed for decision making, it is not always necessary to collect it from original sources as the data may already exist to meet the need. Good sources of second-hand data include Statistics Canada, government records and published reports, and town offices. When using secondary source data, students should consider the appropriateness of the source.

This outcome provides an opportunity for students to work with large numbers in context, such as comparing populations (SCO N01).

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Invite students to write a question about preferred types of books that can be answered using first-hand data and explain why first-hand data is your choice. Ask students to explain how and from whom the data could be collected.
- Ask students to write a question about the populations of the cities and towns in Nova Scotia, and have them explain why the question is best answered using second-hand data. Ask them to identify where they can find this data.
- Invite students to work in groups to generate questions for which the data would be collected first-hand and another question where second-hand data would be appropriate.
- Provide students with a set of data and ask them to generate questions that could be answered using the data.
- Ask students to explain the difference between first-hand and second-hand data and to give examples of each.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide questions and have students determine how best to collect the data in order for students to recognize the difference between first-hand and second-hand data.
- Ask students to generate questions that can best be answered using first-hand data. Ask them to describe how that data could be collected.
- Ask students to generate questions that can best be answered using second-hand data. Ask them to describe how that data could be collected.
- Invite students to explore and discuss the relevance of sample size on first- and second-hand data.
- Provide students with examples of first-hand and second-hand data and ask them to identify the type of data. Encourage students to reflect on the meaning of first-hand and second-hand data and record this reflection in their journals. Involve students in a class discussion on the two types of data.

SUGGESTED LEARNING TASKS

- Ask students to provide examples of data that is relevant to themselves, their families, or their community. Ask them to categorize the data, with explanation, as first-hand or second-hand data.
- Ask students to formulate questions that can best be answered using first-hand data (e.g., What game will we play at home tonight?). Students should describe how this data could be collected (e.g., I can survey everyone at home to find out what games everyone wants to play.). Then, invite students to collect data to answer the question.

- Ask students to formulate a question related to self, family, or community, which can best be answered using second-hand data (e.g., Which has the larger population, my community or my uncle's community?). Students should describe how this data could be collected (e.g., find the data on the Statistics Canada website: www.statcan.gc.ca). Invite students to collect data to answer the question.
- Invite students to find examples of second-hand data in print and electronic media, such as newspapers, magazines, and the Internet, and compare different ways in which the data might be interpreted and used (e.g., statistics about health-related issues, sports data, or votes for favourite websites).

SUGGESTED MODELS AND MANIPULATIVES

- examples of data from print and electronic data

MATHEMATICAL LANGUAGE

Teacher	Student
▪ first-hand data ▪ second-hand data	▪ first-hand data ▪ second-hand data

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 529–530
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 572–573
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 320–323
- *Teaching Student-Centered Mathematics, Grades 5–8* (Van de Walle and Lovin 2006), pp. 309–310

Notes

SCO SP02 Students will be expected to construct and interpret double bar graphs to draw conclusions. [C, PS, R, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

SP02.01 Determine the attributes (title, axes, intervals, and legend) of double bar graphs by comparing a given set of double bar graphs.
SP02.02 Represent a given set of data by creating a double bar graph, label the title and axes, and create a legend without the use of technology.
SP02.03 Draw conclusions from a given double bar graph to answer questions.
SP02.04 Identify examples of double bar graphs used in a variety of print and electronic media, such as newspapers, magazines, and the Internet.
SP02.05 Solve a given problem by constructing and interpreting a double bar graph.

Scope and Sequence

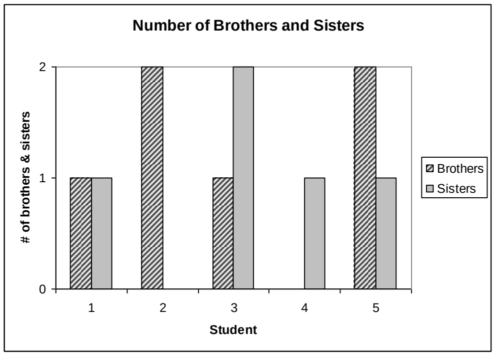
Mathematics 4	**Mathematics 5**	**Mathematics 6**
SP02 Students will be expected to construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.	**SP02** Students will be expected to construct and interpret double bar graphs to draw conclusions.	**SP01** Students will be expected to create, label, and interpret line graphs to draw conclusions. **SP03** Students will be expected to graph collected data and analyze the graph to solve problems.

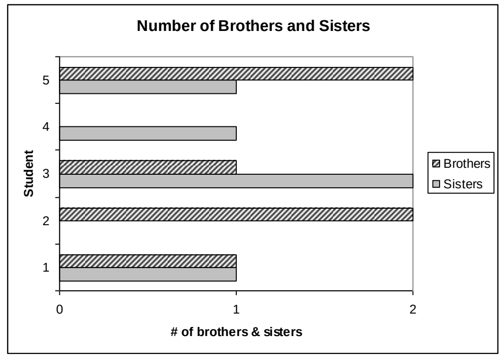
Background

Students should be aware that sometimes when two pieces of data are collected about a certain population, it is desirable to display both sets of data side by side, using the same scale. For example, census data often shows male and female data separately for different years. This is usually done using a **double bar graph**. A **legend** is used to help the reader interpret a double bar graph. An example is presented below. Five students in the class have been asked how many brothers and sisters they have.

	Brothers	Sisters
Student 1	1	1
Student 2	2	0
Student 3	1	2
Student 4	0	1
Student 5	2	1

A double bar graphs allows students to be compared not only in terms of how many brothers they have, or how many sisters they have, but also to compare the number of brothers versus the number of sisters.





Graphs prepared by students are visual communication tools. As such, student-constructed data displays must tell a story. The graph relates the complete story on its own without reference to a written explanation, a table of values, or any other device. It is essential that students include **titles**, **horizontal** and **vertical axis** headings, **scale**, **legends**, and **category labels** in the **legend**. The pairs of bars should be separated by a space from the other pairs of bars, and the order of the bars must remain the same in the graph; that is if the first bar in the first pair of bars appearing on the graph represents data on the number of brothers, then the first bar in all other pairs of bars on the graph must also represent the data about brothers. This is true whether the student draws the graph or constructs it using technology. There are a variety of software and spreadsheet programs available for students to display data quickly and easily. Commercial programs are available to schools through the Nova Scotia School Book Bureau. Other programs are available online. Excel spreadsheets can be constructed using Microsoft Office tools.

Students may benefit from examining graphs with missing labels and posing or answering questions about the graphs. Possible activities include answering given questions, at least some of which are impossible with a label missing: writing an explanation of the graph, which will underscore the communication problem of missing labels critiquing the graph; suggesting improvements that need to be made; and providing suggestions for missing labels so the graph tells a complete story.

A common mistake made by students is to place the numbers on the scale in the space between lines rather than on the place where the line for the limit of that number would be (e.g., 1, 2, etc.).

When students are introduced to a new type of graph, it should be through a context or activity for which this graph would be an ideal way to organize or display the resultant data. Graphs provide opportunities to integrate other mathematics concepts in other strands, such as number and measurement and concepts in other disciplines, such as social studies and science.

Students should have regular opportunities to examine graphs in order to interpret the information displayed, draw conclusions about the data, look for patterns, make predictions, pose questions, and solve problems. Students should also have opportunities to read and interpret graphs found in other sources. Many newspapers use a variety of graphs in their articles and presentations. These can be sources of graphs for discussion and to show how graphs are used in the world around us. Census at School Canada, a project of Statistical Society of Canada, also has a wide range of data displays appropriate for students.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with data. Have them construct a bar graph on grid paper. Ensure that students include a title for the data and labels on both axes.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to describe some data that would be appropriate to display using a double bar graph.
- Ask students to generate a double bar graph from given sets of data without the use of technology.
- Provide students with a double bar graph and have them identify the title, labels, scale, and legend. Ask them to describe why it is important to include each of these for a double bar graph.
- Ask students to construct a double bar graph to help them solve a real-world problem. Ask students to draw one conclusion based on their graph.
- Invite students to draw conclusions from a given double bar graph to answer questions.
  - What information does the graph show?
  - What kinds of data were collected?
  - How many pieces of data were involved?
  - What conclusions can be drawn based on this data?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

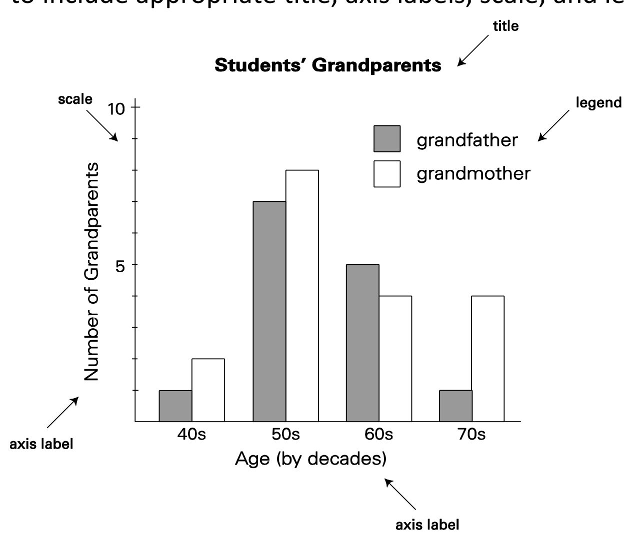
Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to determine when it is appropriate to display data in a double bar graph.
- Provide students with sets of data and have them determine appropriate scales.
- Provide students with two double bar graphs displaying the same data using a different scale, and ask students to determine which graph they prefer and why.
- Invite students to collect first-hand and second-hand data and create double bar graphs making sure to include appropriate title, axis labels, scale, and legend.



- Ask students to use second-hand data collected from sites, such as Statistics Canada or Census at Schools Canada, to create double bar graphs.
- Ask students to interpret a given double bar graph to answer a set of questions.
- Ask students to generate sets of questions that can be answered by reading various double bar graphs.
- Invite students to compare data in the double bar graph within and among the pairs.

- Provide examples of double bar graphs from a variety of media sources, and ask students to bring in examples from similar sources.

SUGGESTED LEARNING TASKS

- Ask students to examine double bar graph samples and determine the attributes (title, axes, legend, intervals). Ask them to compare and share the information displayed.
- Ask students to collect and graph first-hand data, such as girls' and boys' favourite activity in physical education class.
- Invite students to collect information on the length and mass of various animals and display the data in a double bar graph. Ask what conclusions they might draw.
- Invite students to create double bar graphs on subjects that are of personal interest, such as comparing hockey players' salaries from two different teams.

SUGGESTED MODELS AND MANIPULATIVES

- double bar graph samples from various media sources
- graph paper

MATHEMATICAL LANGUAGE

Teacher	Student
▪ create, interpret, and construct ▪ double bar graph, bar(s) ▪ inferences ▪ represent data ▪ title, labels, scale, axis, axes, legend ▪ vertical, horizontal	▪ create, interpret and construct ▪ bar graph, bar(s) ▪ conclusions ▪ represent data ▪ title, labels, scale, axis, axes, legend ▪ vertical, horizontal

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 483–484
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 529–530

Notes

SCO SP03 Students will be expected to describe the likelihood of a single outcome occurring, using words such as **impossible**, **possible**, and **certain**. [C, CN, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

SP03.01 Identify examples of events from personal contexts that are impossible, possible, or certain.
SP03.02 Classify the likelihood of a single outcome occurring in a probability experiment as impossible, possible, or certain.
SP03.03 Design and conduct a probability experiment in which the likelihood of a single outcome occurring is impossible, possible, or certain.
SP03.04 Conduct a given probability experiment a number of times, record the outcomes, and explain the results.

Scope and Sequence

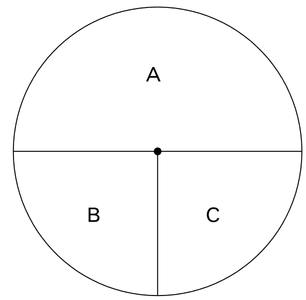
Mathematics 4	**Mathematics 5**	**Mathematics 6**
—	**SP03** Students will be expected to describe the likelihood of a single outcome occurring, using words such as **impossible**, **possible**, and **certain**.	**SP04** Students will be expected to demonstrate an understanding of probability by ▪ identifying all possible outcomes of a probability experiment ▪ differentiating between experimental and theoretical probability ▪ determining the theoretical probability of outcomes in a probability experiment ▪ determining the experimental probability of outcomes in a probability experiment ▪ comparing experimental results with the theoretical probability for an experiment

Background

Mathematics 5 provides an introduction to probability for students. The occurrence of a future event can be characterized along a **continuum** from **impossible** to **certain**. “The key idea to developing **chance** or **probability** on a continuum is to help children see that some events are more likely than others. ... Before students attempt to assign numeric probabilities to events, it is important that they have the basic idea that some events are certain to happen, some are certain not to happen or are impossible, and others have different chances of occurring that fall between these extremes.” (Van de Walle and Lovin 2006b, p. 340).

Students should be encouraged to use their reasoning skills to make predictions about outcomes of events, and to communicate the results using probability language. This introduction to the probability of an event gives students the opportunity to bring their own life experiences to the discussion. Everyday situations should be used as contexts for students to make predictions. Students are expected to tell whether events will always, sometimes, or never occur and to use the terms **impossible**, **possible**, and **certain** to describe those events.

Students are expected to understand that some events are more likely to occur than others. For example, it is more likely that we will have snow in January than in May. It is more likely that we will get a number greater than two, rather than less than two, when we roll a die. Students will realize that although one outcome may be more likely, it may not happen that way in a given set of tries. For example, while it is not very likely, it is possible in the spinner to the right, to spin and land on the B section more often that the A section during a set of 10 spins.



In the probability context, a claim that something will always occur, or never occur, can be refuted with a single counter-example. It might remain true that the event is very likely, or unlikely, but such events have a different status than those which will always or never occur.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Write a journal describing events that are impossible, possible, and certain in our everyday lives.
- Invite students to toss a coin 25 times and record their results in a chart. Ask them to flip the coin another 25 times and record the results. Then, ask students to explain why the results are not the same.
- Ask students to design a spinner in which landing on red is less likely than landing on green, but is more likely than landing on yellow. Ask students to explain their thinking?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

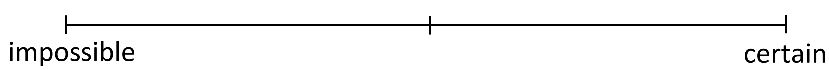
Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide a list of single outcomes (events) ranging from impossible to certain and ask students to identify the likelihood of the event occurring using probability language.
- Invite students to plot outcomes (events) on a probability continuum accompanied by an explanation as to their placement.



- Ask students to generate a list of events that would fall on the probability continuum (e.g., tomorrow is Friday; snow will fall in August in Nova Scotia).
- Invite students to conduct a given probability experiment in which the likelihood of a single outcome occurring is impossible, possible, or certain, recording and explaining their results.
- Invite students to design and conduct probability experiments on a single outcome, recording and explaining their results.

SUGGESTED LEARNING TASKS

- Ask students to describe events that are impossible to occur in the classroom.
- Ask students to describe events that are possible to occur in the classroom.
- Ask students to describe events that are certain to occur in the classroom.

- Invite students to design experiments for which a certain outcome is impossible, possible, or certain.
- Invite students to design a spinner so that the pointer will never land on green.
- Provide opaque bags and coloured cubes for students. Ask them to put 10 cubes in the bags so that it is impossible to choose a red cube. Ask students to repeat this task, but this time putting cubes in the bag so they will are certain red will always be chosen. Finally, ask them to repeat this task so it is possible to choose a red cube.
- As students watch, place five red cubes and five blue cubes in a bag. Then, reach in and pull out a cube. Ask students to explain whether it is certain that you will always pull out that colour cube? Ask them to explain their thinking.
- Invite students to create their own probability events and have other classmates place them
- on the probability line.
- Ask students to explain what will happen if a coin is flipped using the words impossible, possible, or certain.
- Show students a spinner that is completely red. Ask students to explain whether it is possible, impossible, or certain that the spinner will land on red? On green?
- Ask students to design a spinner on which it is impossible to land on blue and certain to land on green.
- Ask students to place cubes in a bag so that it is possible to draw out a cube that is red, blue, or black but impossible to draw out a green cube. Ask them to explain their thinking.

SUGGESTED MODELS AND MANIPULATIVES

- coins
- colour cubes
- colour tiles
- number cubes
- spinners

MATHEMATICAL LANGUAGE

Teacher	Student
▪ experiment ▪ impossible, possible, certain ▪ likelihood ▪ outcomes ▪ probability	▪ experiment ▪ impossible, possible, certain ▪ likelihood ▪ outcomes ▪ probability

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 544–547
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 586–589
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 340–344

Notes

SCO SP04 Students will be expected to compare the likelihood of two possible outcomes occurring, using words such as **less likely**, **equally likely**, or **more likely**. [C, CN, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

SP04.01 Identify outcomes from a given probability experiment that are less likely, equally likely, or more likely to occur than other outcomes.

SP04.02 Design and conduct a probability experiment in which one outcome is less likely to occur than the other outcome.

SP04.03 Design and conduct a probability experiment in which one outcome is equally likely to occur as the other outcome.

SP04.04 Design and conduct a probability experiment in which one outcome is more likely to occur than the other outcome.

Scope and Sequence

Mathematics 4	Mathematics 5	Mathematics 6
—	**SP04** Students will be expected to compare the likelihood of two possible outcomes occurring, using words such as **less likely**, **equally likely**, or **more likely**.	**SP04** Students will be expected to demonstrate an understanding of probability by ▪ identifying all possible outcomes of a probability experiment ▪ differentiating between experimental and theoretical probability ▪ determining the theoretical probability of outcomes in a probability experiment ▪ determining the experimental probability of outcomes in a probability experiment ▪ comparing experimental results with the theoretical probability for an experiment

Background

Once students have mastered the concept of **likelihood** (probability) of a single outcome occurring, they can then begin to compare the likelihood of two outcomes occurring, using the comparative language **less likely**, **equally likely**, and **more likely**.

Students will design and conduct probability experiments for the likelihood of single outcomes occurring, as well as a comparison of two outcomes. They will be expected to record the outcomes and explain the results. Students should have experience using spinners, number cubes, and other concrete materials to gather data that may be used to predict the probability of an outcome.

Games offer many opportunities to place probability in a context. Students should be encouraged to create games and share their ideas with the class.

This outcome is closely connected to outcome SP03.

Additional Information

- See Appendix A: Performance Indicator Background.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Tell students that in a game a player wins if a spinner lands on red, but loses if it lands on blue.
- Ask students to design the spinner for that game. Ask them to explain their thinking.
- Invite students to think of an event that is possible, but not very likely, and another event that is very likely, but might not happen.
- Ask students to describe a game in which it is equally likely that they could win or lose.
- Provide students with a number cube. Ask students to describe the result of rolling a number cube that is
  - less likely to occur than another outcome (e.g., a number less than 3 versus one greater than 3)
  - equally likely to occur (e.g., an even number versus an odd number)
  - more likely to occur than another outcome (e.g., a number less than 5 versus one greater than 5)
- Provide students with a paper bag and tiles or cubes of three different colours (red, blue, and green). Invite students to design and conduct an experiment to determine the probability of choosing a red tile. Ask them to explain their thinking.
- Provide students with a paper bag and 20 colour tiles: 8 blue, 5 green, 5 blue, and 2 yellow. Ask them to describe
  - one outcome that is less likely to occur than other outcomes
  - outcomes that are equally likely to occur
  - one outcome that is more likely to occur than other outcomes

Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

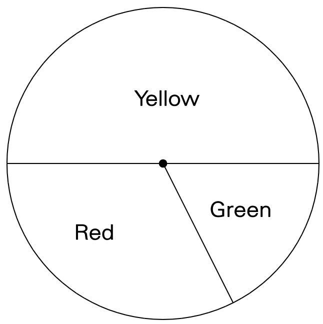
Consider the following strategies when planning daily lessons.

- Give students frequent opportunities to identify outcomes from given probability experiments that are less likely, equally likely, or more likely to occur than other outcomes.
- Invite students to design and conduct probability experiments in which one outcome is less likely, equally likely, and more likely to occur than the other outcome.

Suggested Learning Tasks

- Provide students with a variety of spinner faces. Ask them to determine on which sections of each spinner they are equally likely, more likely, or less likely to land. Spinners may include
  - a spinner with four equal sections labelled with the letters A, B, C, and D
  - a spinner with two sections (one-half each)—one-half should be shaded; one-half should be unshaded
  - a spinner with four sections—Section 1 should be larger than all other sections; Section 2 should be smaller than all other sections; Sections 3 and 4 should be different sizes.
- Ask students to design experiments with two possible outcomes in which one of the outcomes is less likely, equally likely, or more likely to occur.
- Ask students to design spinners such that spinning red is more likely than spinning green, but spinning red is less likely than spinning yellow.
- Ask students to design spinners so that spinning red is more likely than spinning green, but spinning red is less likely than spinning yellow.

- Invite students to design experiments with two possible outcomes in which one of the outcomes is less likely, equally likely, or more likely to occur.
- Ask students to design a spinner so that spinning red is more likely than spinning green, but spinning red is less likely than spinning yellow (see example below).



- Ask students to explain why flipping a coin is considered to be fair.
- Ask students to predict how many heads they will get when they flip a coin 10 times. Ask students to work in pairs to flip a coin 10 times, keeping track of the heads and tails. Share each group's results. Discuss why the results are not the same for all groups. Consider questions such as, Why did heads appear more often for some groups? Why did tails appear more often for some groups? Why didn't everyone get five heads and five tails in their 10 trials?

SUGGESTED MODELS AND MANIPULATIVES

- coins
- colour cubes
- colour tiles
- number cubes
- spinners

MATHEMATICAL LANGUAGE

Teacher	Student
▪ experiment ▪ less likely, equally likely, more likely ▪ likelihood ▪ outcomes ▪ probability	▪ experiment ▪ less likely, equally likely, more likely ▪ likelihood ▪ outcomes ▪ probability

Resources/Notes

Print

- *Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), pp. 544–547
- *Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 586–589
- *Teaching Student-Centered Mathematics, Grades 3–5* (Van de Walle and Lovin 2006), pp. 340–344

Notes

Appendices

Appendix A:
Performance Indicator Background

Number (N)

SCO N01 Students will be expected to represent and partition whole numbers to 1 000 000. [C, CN, V, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N01.01 Read a given numeral without using the word "and."
N01.02 Record numerals for numbers expressed orally, concretely, pictorially, or symbolically as expressions, using proper spacing without commas.
N01.03 Describe the pattern of adjacent place positions moving from right to left.
N01.04 Explain the meaning of each digit in a given numeral.
N01.05 Provide examples of large numbers used in print or electronic media.
N01.06 Express a given numeral in expanded notation.
N01.07 Write the numeral represented by a given expanded notation.
N01.08 Compare and order numbers to 1 000 000 in a variety of ways.
N01.09 Represent a given numeral, 0 to 1 000 000, using a place-value chart.
N01.10 Represent a given number, 0 to 1 000 000, in a variety of ways, and explain how they are equivalent.
N01.11 Represent a given number, 0 to 1 000 000, using expressions.
N01.12 Read and write given numerals, 0 to 1 000 000, in words.

Performance Indicator Background

N01.01 Students must be able to record numbers they hear and read numbers that are written symbolically. Students should read a given numeral (up to one million) without using the word "and." For example, 537 421 is read as five hundred thirty-seven thousand four hundred twenty-one, not as five hundred thirty-seven thousand four hundred and twenty-one. When reading numbers, the word "and" is reserved for the decimal, which will be discussed in outcome N08. Students should also have experience reading numbers in several ways. For example, 938 147 may be read as nine hundred thirty-eight thousand one hundred forty-seven but might also be read as 93 ten thousands, 8 thousands, 1 hundreds, 4 tens, 7 ones or as 938 thousands, 14 tens, 7 ones; or 9 hundred thousands, 37 thousands, 11 hundreds, 3 tens, 17 ones.

N01.02 Students should be given many opportunities to record numbers in symbolic form. Numbers written in standard form are organized and written in groups of three digits. Some authors call each of these groups a **period**. It is not important to highlight the term **period** nor is it intended that students use this term. They can show understanding of the concept without using the term **period**.

Students should write a given numeral using proper spacing without commas. We do not use the comma because in many countries using the SI metric system, the comma is used as the decimal point.

The accepted convention for four-digit numbers is to not leave a space (e.g., 4567). For numbers with five or more digits, leave a small space between each group of three digits starting from the right (e.g., 389 006). If too large a space is used, the number may be misinterpreted as two separate numbers.

When provided with a number represented as a model, expression, expanded notation, place value chart, or words, students need to be able to record the number symbolically in more than one way. For example, if presented with a model or picture of 1 large flat, 2 large rods, 5 large cubes, 2 flats, 3 rods, and 4 small cubes, the number can be recorded in many ways including, 125 234; 100 000, 20 000, 5000, 200, 30, 4; or 1 hundred thousands, 2 ten thousands, 5 thousands, 2 hundreds, 3 tens, 4 ones.

It is important for students to develop a sense of the size of these numbers through concrete and pictorial models. Students should be encouraged to think about how big one million is, or even one billion. Students can construct a concrete representation of some of these numbers. For example, if the large cube represents 1000, students could be asked to imagine a rod that would represent 10 000. They could then use materials such as rolled newspaper and tape to build this rod. They could also build the subsequent flat that represents 100 000, and extend this to building a cube from ten of these flats to make 1 000 000. This 1 000 000 cube will measure one cubic metre, which will make a nice connection to measurement work done in later grades. Students could then speculate whether a 100 000 000 flat would fit inside the classroom. Such constructions help students to develop a conceptual understanding of large numbers.

Students should be able to read large numbers to one-million but should also begin to recognize that the name of a number is connected to the number of digits, for example, that 1 000 000 is one million. This outcome should be embedded in the work done on place-value concepts with emphasis placed on developing students' understanding of large numbers in meaningful contexts.

N01.03 Students should be provided with opportunities to explore place-value patterning for large numbers. When students examine large numbers, they develop a greater sense of the patterning in the place-value system. This exploration will help students to recognize the regularity of the patterns that are inherent in the place-value system. Students should be able to explain that the digits 0–9 are used cyclically to indicate the number of units in any given place. Students should also be able to explain the relationship between each place-value position and its neighbour positions, namely a group of ten in one position makes a group of one in the position to the left, and a group of one in any position makes a group of ten in the position to the right. Students have used this principle to regroup and trade in previous grades, but they should be able to state that this pattern continues to work regardless of the size of the number.

Students should also be given opportunities to examine large numbers in real-world contexts that show the pattern of grouping digits in triads known as periods. They should be able to explain how this patterning facilitates the reading of numbers. For example, 582 582 582 is read as "five hundred eighty-two million, five hundred eighty-two thousand, five hundred eighty-two" since the digits 582 appear in the millions, in the thousands, and in the ones periods. Within each period, there are hundreds, tens and ones. Students need to learn the grouping names (ones, thousands, millions, etc.) to facilitate the reading of numbers, though they will only read numbers to one million. They will recognize that each period has a similar pattern of 100, 10, and 1 of the given period unit. The recognition of this pattern will help students to read larger numbers with which they are unfamiliar.

Students will be expected to be proficient at understanding counting, however may still need some practice with counting through transitions, particularly with large numbers such as ten thousands, hundred thousands, and millions. Counting should be attended to while exploring large numbers and place-value concepts. While developing place-value concepts, students should be provided with opportunities to count on through transitions, (e.g., starting at 999 997 and counting on so that they know that 1 000 000 comes after 999 999).

Students should also be able to connect skip counting by powers of ten to patterns in the place-value system. This will help them to develop an understanding of the patterning of the digits in the place-value system. For example, each place value is ten times the value of the unit to its right. This also extends to the idea that the tens place is 10 times the ones, the hundreds is 100 times the ones, the thousands is 1000 times the ones, and so on.

Thousands (1000)	Hundreds (100)	Tens (10)	Ones (1)
10 × 100 1000 × 1	10 × 10 100 × 1	10 × 1	1

Students should be provided with opportunities to begin with a number such as 90 243 and skip count by 1000 to see that the digit in the thousands place follows the pattern of 0 to 9 and then changes the next place value and begins again as seen below. This regularity of patterning in the place-value system is an essential understanding (e.g., 90 243, 91 243, 92 243, 93 243, ... 99 243, 100 243, ...).

N01.04 Students should learn that the position of a digit determines its value. Students should also recognize and work with the idea that the value of a digit varies, depending on its position or place, in a numeral. Use of the place-value chart can support the development of this understanding. Students should represent given numbers in place-value charts. For example, students would record the number 987 453 in a place-value chart as shown below.

Thousands			**Ones**		
H	T	O	H	T	O
9	8	7	4	5	3

Students should learn that the value of a digit varies, depending on its position or place, in a numeral. Students should recognize the value represented by each digit in a number, as well as what the number means as a whole. The digit “2” in 245 300 represents 2 hundred thousands, whereas the digit “2” in 3200 represents 2 hundreds. Students should be able to explain the meaning of the digits, including numerals with all digits the same (e.g., for the numeral 222 222, the first digit represents 2 hundred thousands, the second digit represents 2 ten thousands, the third digit represents 2 thousands, the fourth digit represents 2 hundreds, the fifth digit represents 2 tens, and the sixth digit represents 2 ones).

It is important to spend time developing a good understanding of the meaning and use of zero in numbers. Students need many experiences using base-ten materials to model numbers with zeros as digits. Teachers should ask students to write the numerals for numbers such as seven thousand five hundred forty or nine thousand two hundred eight. When a number, such as seven hundred four thousand five hundred forty-three, is written in its symbolic form using digits, the digit 0 is called a place holder. If the digit 0 was not used, the number would be recorded as 74 543, and you would mistakenly think that the 7 represented 70 000 instead of 700 000. Students need many experiences using base-ten materials to make connections with the symbols for numbers with zeros as digits.

N01.05 It is important that students view large numbers in print and electronic media in order to see examples of large numbers used in the real world. Newspapers, magazines, e-news, and the Internet provide sources for these large numbers.

N01.06 and **N01.07** Expressions (see indicator N01.11) may also be recorded in the additive expanded form (expanded notation) such as 814 256 is 800 000 + 10 000 + 4000 + 200 + 50 + 6. Expanded form can be demonstrated in either of the following ways:

456 721 = 400 000 + 50 000 + 6000 + 700 + 20 + 1
456 721 = 4 × 100 000 + 5 × 10 000 + 6 × 1000 + 7 × 100 + 2 × 10 + 3 × 1

Students should be exposed to both forms. To foster deep understanding of expanded form, students should be given numbers that include zeros, such as 50 302. Also, expanded form should be given in various orders such as 4 × 10 000 + 3 × 100 000 + 2 × 100.

N01.08 Comparing and **ordering** numbers is fundamental to understanding numbers. Students should investigate meaningful contexts to compare and order two or more numbers, both with and without models. For example, ask them to compare and order populations of communities or capacities of arenas.

Students must realize that when comparing two numbers with the same number of digits, the digit with the greatest place value needs to be addressed first. For example, when asked to explain why one number is greater or less than another, they might say that 251 424 < 367 539 because 251 424 is less than 300 thousands while 367 539 is greater than 300 thousands. When comparing 614 056 and 615 046, students should begin comparing the hundred thousands and then compare each place value to the right. Students must recognize that, when comparing the size of a number, the digit 4 in 432 809 has a greater value than the digit 9, and they should be able to provide an explanation.

Students should not only recognize numbers that are greater or less than a number but also be able to place numbers in between two numbers. For example, which city has a population larger than that of Halifax but smaller than that of Winnipeg?

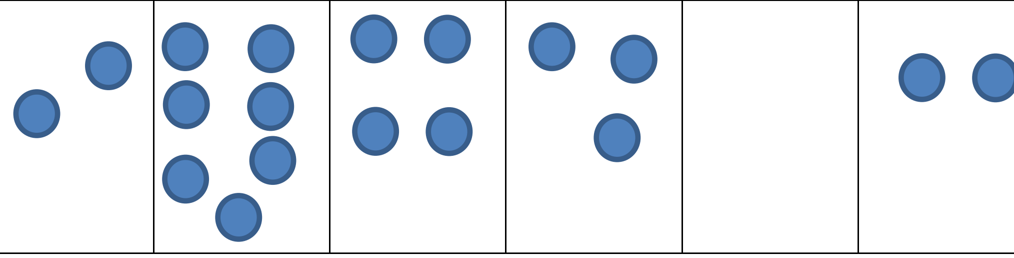
Students should be provided with opportunities to examine large numbers and use place-value arguments to explain which number is larger. Students should also be able to place large numbers in approximate positions on a number line given benchmarks. Teachers should use number lines often and provide opportunities for students to construct various number lines. In previous grades, students have had experiences working with number lines that begin with numbers other than 0 and have a variety of end numbers with and without hatch marks. Students will continue to work with empty number lines and number lines with hatch marks and benchmarks. There are many real-world contexts that can be used to help students make sense of large numbers (e.g., they can be asked to compare and order populations of various countries of the world or different large cities in Canada). Students should also be able to name numbers between any two given numbers; for example, if they know the populations of two large cities, they should be able to identify a population that would fall between these two amounts.

N01.09 Students should use place-value charts to represent numbers to one million.

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones

Students should be presented with a numeral up to 1 000 000 and should use counters to model the number in the place-value chart. For example, if presented with the numeral 274 302, students would record the following:

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones



Students could then write this number in expanded form.

N01.10 Students must have a deep understanding of numbers up to 1 000 000 and be able to represent and rename numbers in a variety of ways. They should be able to translate from one representation to another, for example, from a place-value chart to a numeral or from a base-ten picture to expanded form. Students should be also able to explain why the representations are equivalent. For example, students should be able to represent 129 842 as 129 thousands, 842 ones; as 12 ten thousands, 98 hundreds, 42 ones; or as 12 ten thousands, 9 thousands, 84 tens, 2 ones. Students should be able to explain why each of the representations is equivalent 129 842.

N01.11 Once students have ample opportunities with concrete, pictorial, and verbal representations of base-ten models, they can record the base-ten partitions as an expression such as 793 159 is 700 000 + 93 159. It is important to model the correct use of the term **expression** to students. An expression names a number. Sometimes an expression is a number such as 123 500. Sometimes an expression shows an arithmetic operation, such as 120 000 + 3500 or 124 000 – 500. 123 500 may also be represented by its partitions (parts), such as 100 000 + 23 000 + 500, or 50 000 + 50 000 + 10 000 + 10 000 + 3500. Numbers can also be represented by a difference expression, such as 150 000 – 26 500 or 200 000 – 76 500. Students should also be provided with opportunities to write the numeral represented by a given expression.

Students must have a deep understanding of numbers up to 1 000 000 and be able to rename numbers in a variety of ways. Students should recognize that 1 000 000 is just another expression for 10 hundred thousands, 100 ten thousands, 1000 thousands, 10 000 hundreds, 100 000 tens, and 1 000 000 ones.

N01.12 Students will also need to be able to write the number words for the numbers they encounter and read numbers written in words. The accepted convention for writing number words is as follows

- fifty-six
- three hundred fifty-six
- four thousand three hundred fifty-six
- twenty six thousand nine hundred fifty-six
- one hundred forty-six thousand nine hundred fifty-six
- one million one hundred forty-six thousand nine hundred fifty-six

When students write numbers in words, they must consider the place value of each digit. This solidifies the importance of the periods. For example, to write 946 219 in words, students must recognize that they start with the largest period, in this case thousands, and continue with the successive periods. Students name each period once they say the total number in that period. In 946 219, nine hundred forty-six must be followed with the period name, thousand.

SCO N02 Students will be expected to use estimation strategies, including front-end, front-end adjusted, rounding, and compatible numbers in problem-solving contexts. [C, CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N02.01 Provide a context for when estimation is used to make predictions, check the reasonableness of an answer, and determine approximate answers.
N02.02 Describe contexts in which overestimating is important.
N02.03 Determine the approximate solution to a given problem not requiring an exact answer.
N02.04 Estimate a sum, a difference, a product, or a quotient using an appropriate strategy.
N02.05 Select and explain an estimation strategy for a given problem.

Performance Indicator Background

N02.01 and **N02.03** The ability to estimate computations is a major goal of any modern computational program. For most people in their everyday lives, an estimate is all that is needed to make decisions and to be alert to the reasonableness of numerical claims and answers generated by others and with technology. The ability to estimate rests on a strong and flexible command of facts and mental calculation strategies.

Students should be presented with a variety of problems related to contexts that are meaningful to them. They should be able to transfer the use of operations and estimation strategies to situations found in their daily lives. By using contexts such as measurement, money, and food, students become aware of the use of estimation in their daily lives. The ultimate goal is for students to have a network of estimation strategies that they can flexibly and efficiently apply whenever the need arises.

Van de Walle, in *Elementary and Middle School Mathematics* (2001), suggests that students should be exposed to real examples of estimation in daily life. "Discuss situations where computational estimations are used in real life. Some simple examples include figuring gas mileage, dealing with grocery store situations (doing comparative shopping, determine if there is enough to pay the bill), adding up distances in planning a trip, determining approximate yearly or monthly totals of all sort of things (school supplies, haircuts, lawn-mowing income, time watching TV), and figuring the cost of going to a sporting event or show including transportation, tickets, and snacks. Help children see how each of these involves a computation (in contrast to measurement estimates). ... Discuss why exact answers are not necessary in some instances and why they are necessary in others." (p. 199)

Before attempting pencil-and-paper or calculator computations, students must find estimates, so they are alert to the reasonableness of those pencil-and-paper or calculator answers. Teachers should also model this process before doing any calculations in front of the class. Students should constantly be reminded to estimate before calculating.

While teaching estimation strategies, it is important to use the language of estimation. Some of the common words and phrases are *about*, *just about*, *between*, *a little more than*, *a little less than*, *close*, *close to*, and *near*. It is also important for students to hear and see a variety of contexts for each estimation strategy, so they are able to transfer the use of estimation and strategies to situations found in their daily lives.

N02.02 Brainstorm with students to create a list of real-life situations in which overestimating would be required or desirable. These situations often include estimating the amount of time required to complete certain tasks, estimating whether one has enough money to make a purchase, and estimating quantities of food needed for a specific event. For example, You are in a grocery store with $15 in your wallet. You need to buy milk ($3.98), bread ($2.29), eggs ($2.25), and some meat for supper ($8.75). Will you have enough money at the cash register to buy your food items? If you use front-end estimation, you would estimate a total cost of the groceries to be $3 + $2 + $2 + $8 which is $15. You would conclude that you have enough money, which is not correct. Another example of the importance of overestimating could be planning a party. We often overestimate the amount of food needed to make sure all guests have enough to eat.

N02.04 and **N02.05** Estimation Strategies:

Rounding
This most common estimation strategy involves rounding one, or both, numbers to their highest place values, or to compatible numbers, so the calculation is more easily done mentally. Rounding numbers to the highest place value enables students to keep track of the rounded numbers and do the calculation in their heads using basic facts; however, rounding to two highest place values would require most students to record the rounded number(s) before performing the calculation mentally. Rounding to one or to two highest place values depends upon how close your estimate needs to be to the actual answer.

If the digit before the highest place value to which you are rounding is (a) less than 5, you would normally round down; (b) greater than 5, you would normally round up; (c) exactly a 5, you would round up or down depending upon the overall effect it would have on the answer. However, the decision to round up or to round down should be based upon the effect the rounding will have in the overall calculation. For example, if both numbers to be added have a 5, 50, 500, or 5000, rounding one number *up* and one number *down* will minimize the effect the rounding will have on the estimation. Also, if both numbers are close to 5, 50, 500, or 5000, it may be better to round one up and one down.

Examples:

- In addition, if both addends have a 5 to be rounded, it is best to round one up and one down to minimize the effect of rounding. This is also true if both numbers are close to a 5; for example 648 + 747 would be rounded to 700 + 700 or 600 + 800. To estimate the sum of 4520 + 4610, think, Since both numbers are close to 5000, it would be best to round them to 4000 and 5000, and add to get 9000.
- In subtraction, on the other hand, if both the minuend and subtrahend have 5s to be rounded, both numbers should be rounded up because you are looking for the difference between the two numbers; therefore, you don't want to increase this difference by rounding one up and one down. (This will require careful introduction for students to be convinced. So often students only associate subtraction with *take-away* and need to be reminded that subtraction also finds the *difference* between two numbers. Help them make the connection to the Balancing-for-a-Constant-Difference strategy in mental calculation for subtraction.) To estimate 6237 – 2945, think, 6237 rounds to 6000 and 2945 rounds to 3000; so, 6000 subtract 3000 is 3000.
- To estimate 5549 – 3487, think, Parts of both numbers are close to 500, so round both up. Then 6000 subtract 4000 is 2000.

When rounding multiplication questions with two two-digit factors, round as usual, but if the ones digits are both 5 (or more), consider rounding the smaller factor up and the larger factor down to give a more accurate estimate. For example, 65 × 45 done with a conventional rounding rule would be 70 × 50 = 3500, which would not be close to the actual product of 2845. Using the rounding strategy above, the 45 would round to 50 and the 65 would round to 60, giving an estimate of 3000, much closer to the actual product. (When both numbers would normally round up, the above rule does not hold true.) Students should be encouraged to discuss the closeness of estimates using various methods.

- To estimate 43 × 78, think, Round 43 to 40 and 78 to 80, so 40 × 80 is an estimate of 3200. **(Note:** This uses usual rounding strategies.)
- To estimate 8 × 6930, think, 6930 rounds to 7000, so 8 times 7000 is an estimate of 56 000.
- To estimate 25 × 65, think, Round 25 (the smaller factor) to 30 and 65 to 60, so 30 × 60 is an estimate of 1800. (**Note:** This is a closer estimate than 20 × 60, 20 × 70, or 30 × 70.)
- To estimate 76 × 36, think, 76 (the larger number) rounds down to 70 and 36 (the smaller number) rounds up to 40, so 70 × 40 is an estimate of 2800. (**Note:** This is a closer estimate than 80 × 40 or 80 × 30.)

Students should also use rounding with division questions with a one-digit divisor and a two- or three-digit dividend. Some strategies for estimating division include the following:

- Round one or both numbers to the nearest multiple of 10 or 100. For example,
  - to estimate 78 ÷ 4, students could round 78 to a multiple of 10 and think 80 ÷ 4 = 20
  - to estimate 829 ÷ 4 students could round 829 to a multiple of 100 and think 800 ÷ 4 = 200
  - to estimate 786 ÷ 9, students could round 9 to 10 and 786 to 790, and think 790 divided by 10 is 79, so the solution should be about 79
- Round numbers so that familiar facts can be used. This strategy involves rounding the dividend to a number related to a factor of the divisor and then determining in which place value the first digit of the quotient belongs, to get a “ball-park” answer. Such estimates are adequate in many circumstances. For example,
  - to estimate 643 ÷ 8, students could round 643 to 640, a compatible with 8 in division, and think 640 ÷ 8 = 80
  - to estimate 786 ÷ 9, students could round 786 to 810, a compatible with 9 in division, and then think 810 ÷ 9 = 90
- Round both numbers up or down. For example, to estimate 372 ÷ 9 students could round 372 to 400 and 9 to 10 and think 400 ÷ 10 = 40.

Students should focus on the accuracy of their estimates by considering what happens when they use each of the strategies. They should consider why each change in the dividend and divisor makes sense. Students should be encouraged to estimate before calculating quotients to check the reasonableness of their answer.

To estimate products and quotients efficiently and accurately, students should have quick recall of basic multiplication and division facts and must know how to multiply with multiples of 10, 100, and 1000 and divide with multiples of 10 and 100.

Front-End Estimation

This strategy is the simplest of all the estimation strategies for addition, subtraction, and multiplication. It involves using only the digits in the highest place value of each number to get an estimate. As such, these calculations will only require the use of the basic facts. While this strategy may be applied to division questions if the divisor is a factor of the highest place value of dividend, division estimation is better done by a rounding strategy.

These front-end estimates are adequate in many circumstances, particularly before using a calculator. For addition and multiplication, the actual answer will always be more than the front-end estimates since the digits in the other place values are disregarded. In subtraction, without considering the other digits, you don't know if the actual answer is more, or less, than the front-end estimate.

Examples:

- To estimate 37 260 + 28 142, think, 30 000 plus 20 000 is 50 000, so the estimate is about 50 000.
- To estimate 58 123 – 22 144, think, 50 000 subtract 20 000 is 30 000, so the estimate is about 30 000.
- To estimate 8 × 8548, think, 8 times 8000 is 64 000, so the estimate is about 64 000.
- To estimate 36 × 82, think, 30 times 80 is 2400, so the estimate is about 2400.
- To estimate 625 ÷ 3, think, 600 divided by 3 is 200, so the estimate is about 200.

Adjusted Front-End Estimation

This strategy is often used as an alternative to rounding to get closer estimates. It involves getting a front-end estimate and then adjusting that estimate to get a better, or closer, estimate by either clustering all the values in the other place values to determine whether there would be enough together to account for an adjustment or considering the second-highest place values. This second method of adjustment often results in a closer estimate than first method and would likely only be bettered by the strategy of rounding to the two highest place values.

Examples:

- To estimate 3297 + 2285, think, 3000 plus 2000 is 5000, and 200 plus 200 is only 400, which is not close to another 1000; so, the estimate is 5000. However, clustering 297 and 285 would suggest about 600, so another 1000 would be added to give an estimate of 6000.
- To estimate 5674 – 2487, think, 5000 subtract 2000 is 3000, and 600 – 400 is 200, which is not close to another thousand; so, the estimate stays at 3000 or to estimate 5674 – 2487, think, 5000 subtract 2000 is 3000, and "eyeballing" 674 – 487 suggests there is not another thousand; so, the estimate stays at 3000.
- To estimate 41 679 + 25 342, think, 40 000 plus 20 000 is 60 000, but eyeballing 1679 and 5342 suggests another 10 000, so the adjusted estimate is 60 000 + 10 000, or 70 000; or think, 40 000 plus 20 000 is 60 000 and 1000 plus 5000 is 6000, so the estimate is 60 000 + 6000, or 66 000.

This adjusted front-end strategy is often used as an alternative to rounding to get closer estimates for multiplication questions. It involves getting a front-end estimate and then adjusting that estimate to get a better, or closer, estimate by considering the second-highest place values. This method can be applied to multiplication questions to bypass any consideration of rounding. The two digits to be multiplied will always be visible in the given question, so there will no need to record digits as might occur in the case of rounding to two digits.

Examples:

- To estimate 425 × 32, think, 400 × 30 = 12 000. Then, further adjust the estimate by considering the second-place value and multiply 20 × 30 = 600 for a closer combined estimate of 12 600.
- To estimate 2357 × 6, think, 2000 times 6 is 12 000 and 300 times 6 is 1800, so the estimate is 12 000 + 1800, or 13 800.

Clustering of Near Compatibles

When estimating the addition of a list of numbers, it is sometimes useful to look for two or three numbers that can be grouped to almost make 10s, 100s, or 1000s (compatible numbers). These pairs or trios of numbers provide estimates for 100 or 1000, and are combined with other such estimates, as well as estimates of any leftovers, to get a total estimate for the list.

- For 44 + 33 + 62 + 71, think, 44 plus 62 is close to 100 and 33 plus 71 is close to 100, so the estimate is 100 + 100, or 200.
- For 692 + 604 + 298, think, 692 plus 298 is almost 1000, so the estimate is 1000 + 600, or 1600.
- For 208 + 489 + 812 + 529 + 956, think, 208 and 812 is about 1000, 489 and 529 is about 1000, and 956 is almost another 1000; so the estimate is about 1000 plus 1000 plus 1000, or 3000.
- For 612 – 289 + 397, think, 612 and 397 is about 1000, and 1000 subtract about 300 gives an estimate of 700.

SCO N03 Students will be expected to describe and apply mental mathematics strategies and number properties to recall, with fluency, answers for basic multiplication facts to 81 and related division facts. [C, CN, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N03.01 Describe the mental mathematics strategy used to determine basic multiplication or division facts.
N03.02 Explain why multiplying by 0 produces a product of 0 (zero property of multiplication).
N03.03 Explain why division by 0 is not possible or is undefined (e.g., 8 ÷ 0).
N03.04 Quickly recall multiplication facts up to 9 × 9 and related division facts.

Performance Indicator Background

N03.01 and **N03.04** In general, a computational strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until proficiency is achieved, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

INTRODUCING A STRATEGY

The approach to highlighting a computational strategy is to give students an example of a computation for which strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class. If not, the teacher could share the strategy. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modelling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. Discussion should also include a situation for which the strategy would not be the most appropriate and efficient one. Most important is that the logic of the strategy should be well understood before it is reinforced; otherwise, its long-term retention will be very limited.

REINFORCING A STRATEGY

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to ensure maximum participation. At first, time frames should be generous and then narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After most of the students have internalized the strategy, help them integrate it with other strategies they have developed. Do this by providing activities that include a mix of number expressions, for which this strategy and others would apply. Students should complete the activities and discuss the strategy/strategies that could be used; or they could match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

As strategies are reinforced, students should hear and see the teacher use varied terminology associated with each operation, so they do not develop a single word-operation association. Through rich language usage students are able to quickly determine which operation and strategy they should employ. For example, when a student hears, “two groups of five,” “two rows of five,” “the product of two and five,” they should be able to quickly determine that they must multiply 2 and 5, and that an appropriate strategy to do this is the doubling strategy.

Present students with a variety of contexts for each operation in some of the reinforcement activities so they are able to transfer the use of operations and strategies to situations found in their daily lives. By using contexts, the numbers become more real to the students. Contexts also provide opportunities for students to recall and apply other common knowledge that should be well known. For example, when a student hears the question, “How many days in two weeks?” they should be able to recall that there are seven days in a week and that double seven is 14 days.

ASSESSING STRATEGIES

Assessments of computational strategies should take a variety of forms. In addition to the traditional quizzes, teachers should also record any observations made during the reinforcement activities. Students should be asked for oral responses and written explanations of the strategies used. Individual interviews can provide many insights into a students’ thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students’ abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

After students have achieved competency using one strategy, they should be provided with opportunities for them to integrate it with other strategies they have learned. The ultimate goal is for students to have a network of mental mathematics strategies that they can flexibly and efficiently apply whenever a computational situation arises. This integration can be aided in a variety of ways, some of which are described below.

Take every opportunity that arises in regular mathematics class time to reinforce the strategies learned in mental mathematics time. Include written questions in regular mathematics time. This could be as a journal entry, a quiz/test question, part of a portfolio, or other assessment for which students will get individual feedback. Students should be asked to explain how they could mentally compute a given question in one or more ways, to comment on a student response that has an error in thinking, or to generate sample questions that would be efficiently done by a specified strategy.

MULTIPLICATION FACT LEARNING STRATEGIES

Students will have learned these multiplication fact learning strategies in Mathematics 4. The strategies are included here for review and for reinforcement if students do not have quick recall of their multiplication facts.

The Twos Facts (Doubles)
This strategy involves connecting the addition doubles to the related "two-times" multiplication facts. It is important to make sure students are aware of the equivalence of commutative pairs (2 × ? and ? × 2); for example, 2 × 7 is the double of 7 and that 7 × 2, while it means 7 groups of 2, has the same answer as 2 × 7. When students see 2 × 7 or 7 × 2, they should think, 7 and 7 are 14. Flash cards displaying the facts involving 2 and the times 2 function on the calculator are effective reinforcement tools to use when learning the multiplication doubles.

It is suggested that 2 × 0 and 0 × 2 be left until later when all the zeros facts will be done.

Examples:

- For 2 × 9, think, This is 9 plus 9, so the answer is 18.
- For 6 × 2, think, This 6 plus 6, so the answer is 12.

If students are proficient with doubling, the fours facts (repeated doubling), rather than the Nifty Nines, might be the next strategy explored.

In Mathematics 4, students will have become proficient at **doubling** (e.g., 4 × 3 = (2 × 3) × 2). This idea is extended in Mathematics 5 to include **repeated doubling**. For example, to solve 8 × 6, students can think 2 × 6 = 12 and 4 × 6 = 24, so 8 × 6 = 48. The same principle applies to **halving** and **repeated halving**. For example, for 36 ÷ 4, think 36 ÷ 2 = 18; so 18 ÷ 2 = 9.

The Nifty-Nine Facts
The introduction of the facts involving nines should concentrate on having students discover two patterns in the answers; namely, the tens' digit of the answer is one less than the number of 9s involved, and the sum of the ones' digit and tens' digit of the answer is 9. For example, for 6 × 9 = 54, the tens' digit in the product is one less than the factor 6 (the number of 9s) and the sum of the two digits in the product is 5 + 4 or 9. Because multiplication is commutative, the same thinking would be applied to 9 × 6. Therefore, when asked for 3 × 9, think, The answer is in the 20s (the decade of the answer) and 2 and 7 add to 9; so, the answer is 27. Help students master this strategy by scaffolding the thinking involved; that is, practise presenting the multiplication expressions and just asking for the decade of the answer; practise presenting students with a digit from 1 to 8 and asking them the other digit that they would add to the digit to get 9; and conclude by presenting the multiplication expressions and asking for the answers and discussing the steps in the strategy.

Another strategy that some students may discover and/or use is a compensation strategy, where the computation is done using 10 instead of 9 and then adjusting the answer to compensate for using 10, rather than 9. For example, for 6 × 9, think, 6 groups of 10 is 60 but that is 6 too many (1 extra in each group), so 60 subtract 6 is 54. This strategy can be modelled nicely using ten-frames. Students can build six sets of nine on ten-frames and see that they have almost six full ten-frames (60) but each ten-frame has one counter missing (six less than 60) so there are 54 counters in all. This model can help them to visualize multiples of nine and make sense of this compensation strategy.

While 2 × 9 and 9 × 2 could be done by this strategy, these two nines facts were already handled by the twos facts. This nifty-nine strategy is probably most effective for factors 3 to 9 combined with the factor 9, leaving the 0s and 1s for later strategies.

Examples:

- For 5 × 9, think, The answer is in the 40s, and 4 and 5 add to 9, so 45 is the answer.
- For 9 × 9, think, The answer is in the 80s, and 8 and 1 add to 9, so 81 is the answer.

The Fives Facts

Many students probably have been using a skip-counting-by-5 strategy when 5 has been a factor; however, this strategy is not always the quickest for all combinations, and often results in students' using their fingers to keep track. Therefore, students need to adopt a more efficient strategy.

If students know how to read the various positions of the minute hand on an analog clock, it is easy to make the connection to the multiplication facts involving 5s. For example, if the minute hand is on the 6 and students know that means 30 minutes after the hour, then the connection to 6 × 5 = 30 is easily made. This is why the Five Facts may be referred to as the "clock facts." This would be the best strategy for students who can proficiently tell time on an analog clock.

Another possible strategy involves the patterns in the products. While most students have observed that the Five Facts have a 0 or a 5 as a ones' digit, some have also noticed other patterns. One pattern is that the ones' digit is a 0 if the number of 5s involved is even or the ones' digit is 5 if the number of 5s involved is odd.

Another pattern is that the tens' digit of the answer is half the numbers of 5s involved, or half the number of 5s rounded down. For example, the product of 8 and 5 ends in 0 because there are eight 5s and the tens' digit is 4 because 4 is half of 8; therefore, 8 × 5 is 40. The product of 7 and 5 ends in 5 because 7 is odd and the tens' digit is 3 because half of 7 rounded down is 3; therefore, 7 × 5 is 35.

While these strategies apply to 2 × 5, 5 × 2, 5 × 9, and 9 × 5, these facts were also part of the twos facts and nines facts. The fives facts involving zeros are probably best left for the zeros facts since the minute-hand approach has little meaning for 0.

Examples:

- For 5 × 8, think, When the minute hand is on 8, it is 40 minutes after the hour, so the answer is 40.
- For 3 × 5, think, When the minute hand is on 3, it is 15 minutes after the hour, so the answer is 15.

The Ones Facts

Multiplying by 1 is unique, 1 × ___ simply means one group of ___. While the ones facts are the "no change" facts, it is important that students understand why there is no change. On a number line students can see that 1 hop of 3 moves them to 3. When building sets, one set of 5 is 5. Many students get these facts confused with the addition facts involving 1. To understand the ones facts, knowing what is happening when we multiply by one is important. For example 6 × 1 means six groups of 1 or 1 + 1 + 1 + 1 + 1 + 1 and 1 × 6 means one group of 6. It is important to avoid teaching arbitrary rules such as "any number multiplied by one is that number." Students will come to this rule on their own given opportunities to develop understanding. Be sure to present questions visually and orally; for example, "4 groups of 1" and 4 × 1, and "1 group of 4" and 1 × 4.

While this strategy applies to 2 × 1, 1 × 2, 1 × 5, and 5 × 1, these facts have also been handled previously with the other strategies.

Examples:
- For 8 × 1, think, Eight 1s make 8.
- For 1 × 7, think, One 7 is 7.

The Zeros Facts
As with the ones facts, students need to understand why these facts all result in zero because they are easily confused with the addition facts involving zero; thus, the zeros facts are often “tricky.” To understand the zeros facts, students need to be reminded what is happening by making the connection to the meaning of the number sentence. For example, 6 × 0 means “six 0s” or “six sets of nothing.” This could be shown by drawing six boxes with nothing in each box. 0 × 6 means “zero sets of 6.” This is much more difficult to conceptualize; however, if students are asked to draw two sets of 6, then one set of 6, and finally zero sets of 6, where they don’t draw anything, they will realize why zero is the product. Similar to the previous strategy for teaching the ones facts, it is important not to teach a rule such as “any number multiplied by zero is zero.” Students will come to this rule on their own, given opportunities to develop understanding.

Examples:
- For 7 × 0, think, Having 7 zeros means having a total of zero.
- For 0 × 8, think, Having no 8s means having zero.

The Threes Facts (double plus one more set)
The way to teach the threes facts is to develop a “double plus one more set” strategy. Invite students to examine arrays with three rows. If they cover the third row, they easily see that they have a “double” in view; so, adding “one more set” to the double should make sense to them. For example, for 3 × 7, think, 2 sets of 7 (double) plus one set of 7 or (7 × 2) + 7 = 14 + 7 = 21. This strategy uses the doubles facts that should be well known before this strategy is introduced; however, there will need to be a discussion and practise of quick addition strategies to add on the third set.

While this strategy can be applied to all facts involving 3, the emphasis should be on 3 × 3, 3 × 4, 4 × 3, 3 × 6, 6 × 3, 3 × 7, 7 × 3, 3 × 8, and 8 × 3, all of which have not been addressed by earlier strategies.

Examples:
- For 3 × 6, think, Two 6s make 12, plus one more 6 is 18.
- For 4 × 3, think, Two 4s make 8, plus one more 4 is 12.

The Fours Facts (Repeated Doubling)
The way to teach the fours facts is to develop a “double-double” strategy. Invite students to examine arrays with four rows. If they cover the bottom two rows, they easily see they have a “double” in view and another “double” covered; so, doubling twice should make sense. For example, for 4 × 7, think, 2 × 7 (double) is 14 and 2 × 14 is 28. Discussion and practice of quick mental strategies for the doubles of 12, 14, 16, and 18 will be required for students to master their fours facts. (One efficient strategy is front-end whereby the ten is doubled, the ones are doubled, and the two results are added together. For example, for 2 × 16, think, 2 times 10 is 20, 2 times 6 is 12, so 20 and 12 is 32.)

While this strategy can be applied for all facts involving 4, the emphasis should be on 4 × 4, 4 × 6, 6 × 4, 4 × 7, 7 × 4, 4 × 8, and 8 × 4, all of which have not been addressed by earlier strategies.

Examples:
- For 4 × 6, think, Double 6 is 12, and double 12 is 24.
- For 8 × 4, think, Double 8 is 16, and double 16 is 32.

The Last Nine Facts

After students have worked on the above seven strategies for learning the multiplication facts, there are only nine facts left to be learned. These include 6 × 6, 6 × 7, 6 × 8, 7 × 7, 7 × 8, 8 × 8, 7 × 6, 8 × 7, and 8 × 6. At this point, the students themselves can probably suggest strategies that will help with quick recall of these facts. Each fact may be presented to students and then they can be asked for their suggestions. Among the strategies suggested might be one that involves decomposition and the use of helping facts such as skip counting up or down from a known fact. This reinforces the meanings of multiplication (and division) as students must be thinking about the addition or subtraction of "groups." For example, for 8 × 7, think 7 × 7 = 49 and then add another group of 7; 49 + 7 = 56.

The distributive property may also be helpful in learning these or other facts. The distributive property relates to the fact that sets can be broken down into subsets, for example 5 sets of 3 can be thought of as

- 4 sets of 3 and 1 set of 3
- 3 sets of 3 and 2 sets of 3
- 5 sets of 2 and 5 sets of 1

Understanding this principle will help students learn the multiplication and division facts (e.g., 6 × 8 can be thought of as 5 × 8 + 1 × 8; or 36 ÷ 6 as 30 ÷ 6 + 6 ÷ 6. Students learn division facts by thinking about corresponding multiplication facts. They can reduce the number of separate multiplication facts to be learned by drawing on a relationship previously explored (e.g., any multiple of 4 is twice the same multiple of 2). To help students learn to determine one fact based on what they know about another, include, on a regular basis, questions such as, How does knowing 5 × 4 = 20 help you to know 6 × 4? or What other division fact could help you solve 48 ÷ 6?

The distributive property is illustrated by

XXXXX	XXX	4 × 8 = (4 × 5) + (4 × 3)
XXXXX	XXX	= 20 + 12
XXXXX	XXX	= 32
XXXXX	XXX	

Examples:

- For 6 × 6, think, five sets of 6 is 30 plus one more set of 6 is 36.
- For 6 × 7 or 7 × 6, think, five sets of 6 is 30 plus two more sets of 6 is 12, so 30 plus 12 is 42.
- For 6 × 8 or 8 × 6, think, five sets of 8 is 40 plus one more set of 8 is 48. Another strategy is to think, three sets of 8 is 24 and double 24 is 48.
- For 7 × 7, think, five sets of 7 is 35, two sets of 7 is 14, so 35 and 14 is 49. (This is more difficult to do mentally than most of the others; however, many students seem to commit this one to memory quite quickly, perhaps because of the uniqueness of 49 as a product.)
- For 7 × 8, think, five sets of 8 is 40, two sets of 8 is 16, so 40 plus 16 is 56. (Some students may notice that 56 uses the two digits 5 and 6 that are the two counting numbers before 7 and 8.)
- For 8 × 8, think, four sets of 8 is 32, and 32 doubled is 64. (Some students may know this as the number of squares on a chess or checker board).

DIVISION FACT LEARNING STRATEGIES

It could be argued that there is no such thing as division facts. When faced with a division question, most people scan their memories for the related multiplication fact. For example, if asked to find 36 ÷ 4, they ask themselves, 4 times what number is 36? In fact, this is the think-multiplication strategy that would have been introduced when the division concept was introduced and reinforced in the regular classroom. This strategy, however, rests on a thorough knowledge and a quick recall of the multiplication facts. Therefore, students must have mastered their multiplication facts before they attempt to get a three-second, or less, response to the division facts.

Arrays and sets are important in helping students establish the relationship between multiplication and division and in the development of computational procedures for multiplication and division. Coloured tiles are effective when exploring arrays for this purpose. Using coloured tiles on an overhead, have a class discussion to create as many rectangles as possible that have 12 square units. Relate each of the rectangles to multiplication facts, (1 × 12, 2 × 6, and 3 × 4). Next, split each rectangle into equal groups of coloured tiles to develop the corresponding division facts. (12 ÷ 1 = 12, 12 ÷ 12 = 1, 12 ÷ 2 = 6, 12 ÷ 6 = 2, 12 ÷ 4 = 3, 12 ÷ 3 = 4)

To help students achieve a three-second, or less, response time for the division facts, the facts may be organized into clusters rather than try to reinforce all the facts at once. These clusters can relate to the corresponding multiplication fact clusters. For example, the 17 non-zero Twos Facts in multiplication have 17 corresponding division facts: 18 ÷ 2, 18 ÷ 9, 16 ÷ 2, 16 ÷ 8, 14 ÷ 2, 14 ÷ 7, 12 ÷ 2, 12 ÷ 6, 10 ÷ 2, 10 ÷ 5, 8 ÷ 2, 8 ÷ 4, 6 ÷ 2, 6 ÷ 3, 4 ÷ 2, 2 ÷ 2, 2 ÷ 1. Discuss these related facts; reinforce them until the three-second, or less, response time is achieved.

Introduce and reinforce the cluster of 15 new related Nifty-Nines division facts (18 ÷ 2 and 18 ÷ 9 were already in the first cluster) until the three-second response time for these facts is achieved; and integrate and reinforce these 32 facts. Continue isolating and reinforcing other clusters and integrating them until students have achieved a three-second, or less, response time for the full set of division facts.

Because 19 of the 100 multiplication facts involve zero and division by zero is undefined, there are only 81 corresponding division facts.

Possible cluster of division facts:
- From The Twos Facts in Multiplication (17): 18 ÷ 2, 18 ÷ 9, 16 ÷ 2, 16 ÷ 8, 14 ÷ 2, 14 ÷ 7, 12 ÷ 2, 12 ÷ 6, 10 ÷ 2, 10 ÷ 5, 8 ÷ 2, 8 ÷ 4, 6 ÷ 2, 6 ÷ 3, 4 ÷ 2, 2 ÷ 2, 2 ÷ 1
- From The Nifty-Nines Facts in Multiplication (15): 81 ÷ 9, 72 ÷ 9, 72 ÷ 8, 63 ÷ 9, 63 ÷ 7, 54 ÷ 9, 54 ÷ 6, 45 ÷ 9, 45 ÷ 5, 36 ÷ 9, 36 ÷ 4, 27 ÷ 9, 27 ÷ 3, 9 ÷ 9, 9 ÷ 1
- From The Fives Facts in Multiplication (13): 40 ÷ 5, 40 ÷ 8, 35 ÷ 5, 35 ÷ 7, 30 ÷ 5, 30 ÷ 6, 25 ÷ 5, 20 ÷ 5, 20 ÷ 4, 15 ÷ 5, 15 ÷ 3, 5 ÷ 5, 5 ÷ 1
- From The Ones Facts in Multiplication (11): 8 ÷ 1, 8 ÷ 8, 7 ÷ 1, 7 ÷ 7, 6 ÷ 1, 6 ÷ 6, 4 ÷ 1, 4 ÷ 4, 3 ÷ 1, 3 ÷ 3, 1 ÷ 1
- From The Threes Facts in Multiplication (9): 24 ÷ 3, 24 ÷ 8, 21 ÷ 3, 21 ÷ 7, 18 ÷ 3, 18 ÷ 6, 12 ÷ 3, 12 ÷ 4, 9 ÷ 3
- From The Fours Facts in Multiplication (7): 32 ÷ 4, 32 ÷ 8, 28 ÷ 4, 28 ÷ 7, 24 ÷ 4, 24 ÷ 6, 16 ÷ 4
- The Last Facts (9): 64 ÷ 8, 56 ÷ 8, 56 ÷ 7, 49 ÷ 7, 48 ÷ 8, 48 ÷ 6, 42 ÷ 7, 42 ÷ 6, 36 ÷ 6

Skip counting from a known fact can be used as a tool for division. For example, if the known fact is 40 ÷ 8 = 5, then use this fact to determine 56 ÷ 8 by skip counting up two more 8s to get from 40, 48, 56, which shows 56 ÷ 8 = 7. Skip counting from a known fact can also work by skipping back. For example, if the known fact is 80 ÷ 8 = 10, then use this fact to determine 72 ÷ 8 by skip counting down one more 8 to get from 80 to 72.

To illustrate the repeated halving method, provide students with the problem 32 ÷ 4. Place 32 counters in an array on the overhead. When the student is unable to divide by 4, they can instead, divide by 2 and do this twice for the same result. That is, 32 ÷ 2 = 16 and 16 ÷ 2 = 8; therefore 32 ÷ 4 = 8.

N03.02 Multiplying by zero is unique, ___ × 0 = 0 since many zeros still equal zero. To help students develop good understandings, use contexts and pictures (e.g., On a number line students can see that three hops of zero or zero hops of three leaves them still on zero). To show five sets of zero you might use five empty baskets and ask, How many muffins are there in all? Since there is nothing (zero muffins) in any of the baskets, the answer is zero, because five groups of zero is zero. It will not matter how many empty baskets there are, any number of baskets with zero muffins in them, result in zero muffins altogether. (5 × 0 = 0).

N03.03 Along with understanding why multiplication by zero produces a product of zero, students must be able to explain why **division by zero is undefined** or **not possible**. It is not possible to make a set of zero from a given group, nor is it possible to make zero sets from a given group. When demonstrated as repeated subtraction, removing groups of zero will never change your dividend (e.g., 20 ÷ 5 = 4 because 20 – 5 – 5 – 5 – 5 = 0). However 5 ÷ 0 is undefined because no matter how many times 0 is subtracted from 5, you will never reach 0 (e.g., 5 – 0 – 0 – 0 – ... = 5, not 0). Rather than telling students these properties, pose problems involving division with zero.

N03.04 Response time is an effective way to see if students have automaticity of their facts. For the multiplication and division facts, the goal is for students to respond in three-seconds or less by the end of the year. Students should be given more time than this in the initial strategy reinforcement activities. The time can be reduced as students become more proficient applying the strategy until the three-second goal is reached. The three-second response goal is a guideline for the teacher and does not need to be shared with students if it will cause undue anxiety.

SCO N04 Students will be expected to apply mental mathematics strategies for multiplication, including ▪ multiplying by multiples of 10, 100, and 1000 ▪ halving and doubling ▪ using the distributive property [C, ME, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N04.01 Determine the products when one factor is a multiple of 10, 100, or 1000.
N04.02 Apply halving and doubling when determining a given product (e.g., 32 × 5 is the same as 16 × 10).
N04.03 Apply the distributive property to determine a given product that involves multiplying factors that are close to multiples of 10 (e.g., 98 × 7 = (100 × 7) – (2 × 7)).

Performance Indicator Background

N04.01

Multiplying by 10, 100, and 1000
This strategy involves keeping track of how the place values have changed as a result of being multiplied by powers of ten. Introduce these products through base-ten block representations. For example, for 10 × 53, display 5 rods and 3 small cubes to represent 53, and think, 10 sets of 5 rods would be 50 rods, or 5 flats, and 10 sets of 3 small cubes would be 30 small cubes, or 3 rods; so, 5 flats and 3 rods represents 530. Through a few similar examples, it should become clear that multiplying by 10 changes the place value of each digit by one position to the left, because the product is ten times as much.

Multiplying by 10 increases, by one place, all of the place values of a number. For 10 × 67, think, the 6 tens becomes 6 hundreds and the 7 ones becomes 7 tens; therefore, the answer is 670.

Multiplying by 100 increases, by two places, all of the place values of a number. For 100 × 86, think, the 8 tens becomes 8 thousands and the 6 ones becomes 6 hundreds; therefore, the answer is 8600. It is necessary that students use the correct language when orally answering questions where they multiply by 100. For example, the answer to 100 × 86 should be read as 86 hundred, which is equivalent to 8 thousand 6 hundred.

Multiplying by 1000 increases, by three places, all of the place values of a number. For 1000 × 45, think, the 4 tens becomes 40 thousands and the 5 ones becomes 5 thousands; therefore, the answer is 45 000. This could also be thought of as 45 ones becoming 45 thousands instead of dealing with the digits separately. It is necessary that students use the correct language when orally answering questions where they multiply by 1000. For example the answer to 1000 × 45 should be read as 45 thousand. Alternatively, students might use a calculator to multiply various numbers by 1000 and should then detect that the pattern of changing place values continues with multiplying by 1000, changing all the place values of a number by three positions to the left because the product is 1000 times as much.

Multiplying by Multiples of 10, 100, and 1000

After the multiplication facts and the related strategies are reviewed, or at the same time, these facts should be extended to the 10s, 100s, and 1000s. Students, however, should be encouraged to approach these questions as modelled in the “think lines” in the examples provided below, so the place value of the answers is known before any multiplication is undertaken. It would be beneficial to connect these products to groups of base-ten blocks. For example, 6 groups of 3 small cubes, 6 groups of 3 rods, 6 groups of 3 flats, and 6 groups of 3 large cubes all result in 18 blocks, whether those blocks represent 1s, 10s, 100s, or 1000s.

For example, to mentally calculate 3 × 70, think, The answer can be expressed in tens and the number of those tens is 3 × 7 or 21, so the answer is 21 tens or 210. For 6 × 900, think, The answer will be hundreds and the number of those hundreds is 6 × 9 or 54, so the answer is 54 hundreds, or 5400. For 4 × 6000, think, The answer will be thousands and the number of those thousands is 4 × 6 or 24, so the answer is 24 thousands, or 24 000.

The multiplication facts should also be applied to the product of two multiples of 10 (less than 100). The strategy for finding the product of two multiples of 10 is to realize that the product will be a number of hundreds and that number of hundreds is found by multiplying the two single-digits in the tens places. For 30 × 80, think, The product of two tens can be expressed in hundreds, and the number of those hundreds is 3 times 8 or 24, so the answer is 24 hundreds, or 2400. For 50 × 70, think, The product of two tens is hundreds, and the number of those hundreds is 5 times 7 or 35 hundreds, so the answer is 35 hundreds or 3500. For 60 × 50, think, The product of two tens is hundreds, and the number of those hundreds is 6 × 5 or 30, so the answer is 30 hundreds, or 3000. To convince students that these questions always result in hundreds, model some products, such as 20 × 30 and 30 × 40, as arrays with base-ten flats. For example, 20 × 30 would be modelled as 2 rows of 3 flats: the dimensions of the rectangle formed by these 6 flats would be 20 by 30; therefore, 20 × 30 is 600.

This strategy should be introduced before students begin pencil-and-paper strategies for products of two two-digit numbers, so they can estimate these products.

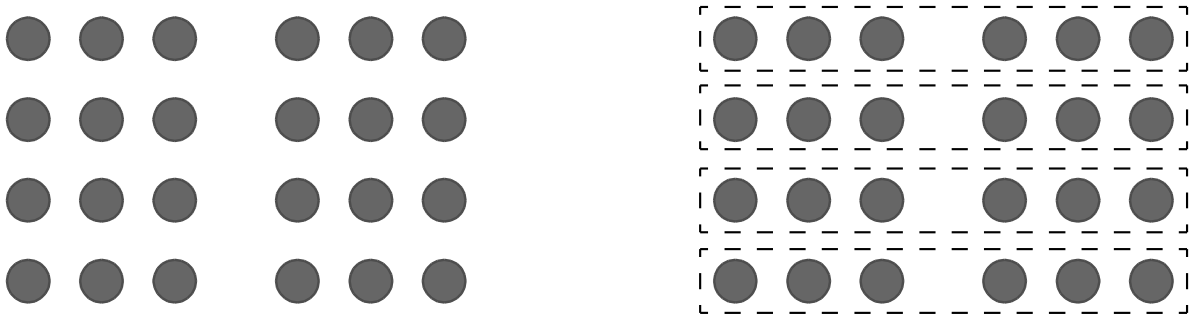
N04.02

The Halving and Doubling

This strategy involves halving one factor and doubling the other factor in order to get two new factors that are easier to calculate. While the factors have changed, the product is equivalent, because multiplying by one-half and then by 2 is equivalent to multiplying by 1, which is the multiplicative identity. Halving and doubling is a situation where students may need to record some sub-steps. For example, to solve 4 × 16, students can think of it as 2 × 32 (halving 4 and doubling 16) or 8 × 8 (doubling 4 and halving 16) which equals 64.

The halving and doubling strategy is a specific example of the multiplication principle that states to multiply two numbers you can divide one factor and multiply the other by the same number without changing the product.

"Consider the 8 groups of 3 (8 × 3) shown below. If you pair up groups of 3, you will have 6 in each group, (twice as many in each group) but only 4 groups (half as many groups), while the total number of circles stays the same (8 × 3 = 4 × 6)."



8 groups of 3 8 × 3 = 4 × 6 4 groups of 6

(Small 2008a, 28)

The halving and doubling strategy works best when one of the factors is even, since having a factor an odd number results in a fraction. This strategy is particularly useful for factors such as 5, 15, 25, etc. For example, 12 × 15 can be thought of as 6 × 30 = 180 and 25 × 18 can be thought of as 50 × 9 = 450.

N04.03

Distributive Property

The ability to partition numbers is important in multiplication. For example, to multiply 5 × 43, think 5 × 40 (200) and 5 × 3 (15) and then add the results. This principle also applies to multiplication questions in which one of the factors is close to a multiple of 10; that is, one of the factors ends in a nine (or eight or seven). For such questions, one could use a **compensating strategy**—multiply by the next multiple of ten and compensate by subtracting to find the actual product. For example, when multiplying 39 by 7 mentally, one could think, 7 times 40 is 280, but there were only 39 sevens so I need to subtract 7 from 280 which gives an answer of 273. The example given in the indicator combines the distributive property with compensation. It could also be approached in either of the following ways: 98 × 7 = (90 × 7) + (8 × 7) or 98 × 7 = (100 × 7) – (2 × 7).

SCO N05 Students will be expected to demonstrate, with and without concrete materials, an understanding of multiplication (two-digit by two-digit) to solve problems.
[C, CN, PS, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

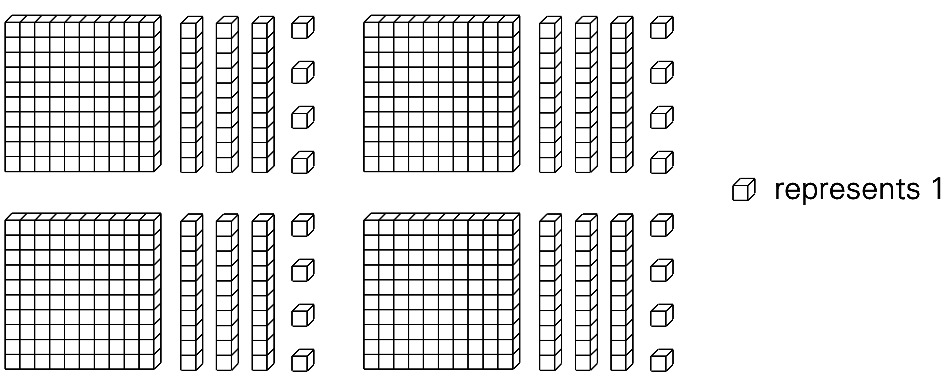
Performance Indicators

N05.01 Model the multiplication of two-digit factors, using concrete and visual representations of the area model, and record the process symbolically.
N05.02 Illustrate partial products in expanded notation for both factors (e.g., For 36 × 42, determine the partial products for (30 + 6) × (40 + 2).).
N05.03 Represent both two-digit factors in expanded notation to illustrate the distributive property (e.g., To determine the partial products of 36 × 42, record
(30 + 6) × (40 + 2) = 30 × 40 + 30 × 2 + 6 × 40 + 6 × 2 = 1200 + 60 + 240 + 12 = 1512.).
N05.04 Describe a solution procedure for determining the product of two given two-digit factors, using a pictorial representation such as an area model.
N05.05 Solve a given multiplication problem in context, using personal strategies, and record the process.
N05.06 Create and solve multiplication story problems, and record the process symbolically.
N05.07 Determine the product of two given numbers using a personal strategy and record the process symbolically.

Performance Indicator Background

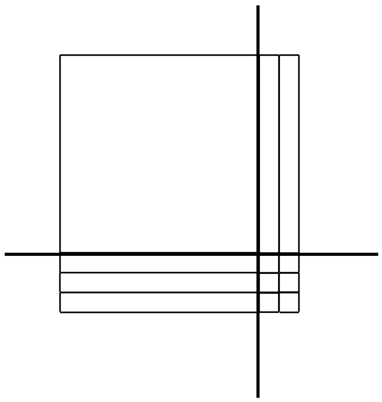
N05.01, **N05.02**, **N05.03**, and **N05.04** In Mathematics 3 and 4, students learned that multiplication and division can be modelled with sets, arrays, area models, and number lines. They should continue to use these models in determining products in Mathematics 5.

For example, a student may model 4 × 134 using a set model represented with base-ten materials to develop a strategy based on the distributive property and partial products.



4 × 100 = 400
4 × 30 = 120
4 × 4 = 16
400 + 120 + 16 = 536
4 × 134 = 536

Students may also model the multiplication of two two-digit numbers as the **area** of a rectangle with the dimensions of the two numbers. Related discussions should focus on the expanded notation of the two factors and the four partial products that are evident in the four regions of the area model. This will help students to develop a variety of strategies that make sense.

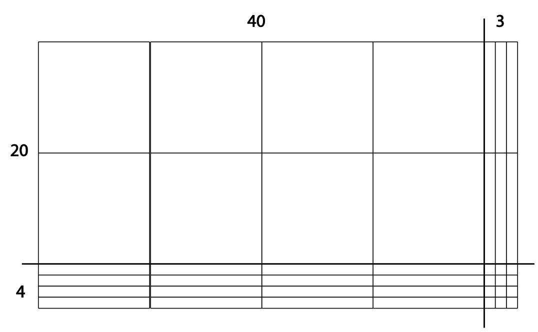


13 rows of 12
10 × 10 = 100
3 × 10 = 30
10 × 2 = 20
3 × 2 = 6
13 × 12 = 156

Students should relate the model to an **algorithm**. The symbolic steps should be recorded and related to each physical manipulation. When students understand the **area model**, they may choose to use a grid-paper drawing as an explanation, but it is important to record the process symbolically. A standard algorithm might be presented, but it is important that an explanation with models be provided, not just procedural rules.

The **commutative property** of multiplication means the order in which you multiply does not change the product. This is sometimes helpful in rearranging the factors to make a calculation “friendlier.” The **distributive property** of multiplication allows students to record **partial products**. For example,

24 × 43 = (20 + 4) × (40 + 3)
20 × 40
20 × 3
4 × 40
4 × 4



The above example closely resembles what students may already think of as **front-end multiplication**. Students should be given the opportunity to use various algorithms. If, however, students are using inefficient algorithms, they should be guided to select more appropriate ones. It is helpful for students to be exposed to various algorithms and for them to invent their own strategies. One algorithm may be more meaningful to a student than another or one algorithm may work better for a particular set of numbers. It is important for students in this grade to refine their personal strategies to increase their efficiency.

Students should be able to represent both two-digit factors in expanded notation. They may use the expanded notation and the distributive property to determine partial products. For example, to calculate 45 × 18 the student should record (40 + 5) × (10 + 8). Using the distributive property, the students would then record (40 × 10) + (40 × 8) + (5 × 10) + (5 × 8). Calculating partial products, the student would record 400 + 320 + 50 + 40, and would then calculate the final product as 810.

Quick Multiplication

This pencil-and-paper strategy is applied to questions in which there are more than two sub-products but with no regrouping needed. In Mathematics 5, the questions will involve two- and three-digit numbers multiplied by single-digit numbers. The questions are usually presented visually rather than orally, and students always record their answers on paper. This strategy partially relies on mental mathematics because students should do all the combinations in their heads starting at the front end, recording only the results of each sub-product. Most likely, students will simply multiply the digits of the number, starting at the front end, without being conscious of the place-value names; therefore, when you call upon them to state their answers, insist that they read the number using correct place-value terminology. In any discussion of the strategy, be sure to use the place-value names as shown in the "think line" in the examples provided. For example, to calculate 31 × 3, the student thinks, 3 times 30 is 90, and 3 × 1 is 3; 90 + 3 is 93. To calculate 423 × 2, the student thinks, 2 times 400 is 800, 2 times 20 is 40, 2 times 3 is 6, and 800 + 40 + 6 = 846; or think and record resultant digits: 2 times 4 is 8 (in the hundreds place), 2 times 2 is 4 (in the tens place), and 2 times 3 is 6 (in the ones place), so the answer is 846 (eight hundred forty-six).

N05.05 and **N05.06** For multiplication and division there are three story problem structures: equal groups, comparing, and combining. Equal groups, where the result is unknown, is the typical type of multiplication used in school and is associated with repeated addition. The equal groups structure also involves determining the size of a group (partition division) or the number of equal groups (measurement division). Additionally, multiplication and division are used in comparison situations. Multiplicative comparisons lay the groundwork for proportional reasoning. With comparison, we may have a situation where we want to determine the size of a result given the initial amount and the multiplier (multiplication) or we may wish to determine the size of the initial set or the multiplier (division). The third structure is the combinations structure, which provides the foundation for later work in probability. Combinations have only two sub-structures: determining the product given the size of the two sets, or determining the size of one set given the product and the other set. These structures of story problems have been given special attention here because these situational contexts help students to see the circumstances under which they might use multiplication and division.

<table>
<tr><th>Equal Groups</th><th>Comparison</th><th>Combinations</th></tr>
<tr>
<td>Result Unknown
(Given the number of groups and the size of the group, find the result.)

A bag holds 8 carrots. If you have 5 bags of carrots, how many carrots do you have?

5 × 8 = ?

There are 5 rows of chairs in the library. Each row has 9 chairs in it. How many chairs are in the library?

5 × 9 = ?

A grasshopper jumps 9 cm in a single jump. If the grasshopper jumps 6 times, what distance will it have travelled?

9 × 6 = ?</td>
<td>Result Unknown
(Given the initial amount and the multiplier, find the result.)

Kylie ate 5 apples last week. Her brother ate twice as many apples. How many apples did her brother eat last week?

5 × 2 = ?</td>
<td>Result Unknown
(Given the size of the two sets, find the result.)

Khaled has 3 pairs of pants and 5 shirts. How many different outfits can he make?

3 × 5 = ?</td>
</tr>
<tr>
<td>Size of a Group Unknown
(Given the result and the number of equal groups, find the size of the group.) (partition division)

You have 32 chairs. You need to put them in 8 rows. How many chairs will be in each row?

32 ÷ 8 = ?
or
8 × ? = 32</td>
<td>Multiplier Unknown
(Given the result and the initial amount, find the multiplier.)

A frog jumped 2 metres. A kangaroo jumped 12 metres. How many times farther did the kangaroo jump?

12 ÷ 2 = ?
or
2 × ? = 12</td>
<td rowspan="2">One Set Unknown
(Given the result and one of the sets, find the other set.)

Chika likes to eat yogurt with berries for recess. Chika has 5 different kinds of berries that she adds to her yogurt. If she can make 15 different yogurt with berries snacks, how many different kinds of yogurt does she use to make her snacks?

15 ÷ 5 = ?
or
5 × ? = 15</td>
</tr>
<tr>
<td>Number of Equal Groups Unknown
(Given the result and the size of the set, find the number of groups.) (measurement division)

You have 27 photographs. You want to put 3 photographs on each page of your photo album. How many pages will you fill?

27 ÷ 3 = ?
or
3 × ? = 27</td>
<td>Initial Unknown
(Given the result and the multiplier, find the initial amount.)

Katy collected 45 cans for recycling. That was 5 times as many as cans as Beth collected. How many cans did Beth collect for recycling?

45 ÷ 5 = ?
or
5 × ? = 45</td>
</tr>
</table>

Students should be encouraged to both solve and create problems related to these structures. Students should solve problems by building and sketching models and explaining their discoveries both symbolically and verbally (either written or oral). Students should not be expected to do symbolic manipulation in isolation. Students should be exposed to multiplication and division situations that enable them to understand the various contexts in which we use multiplication and division. Multiplication situations should involve sets, arrays, area models, and linear or measurement models, such as number lines. For division, students should be exposed to situations that involve determining both, how much in each group and how many groups.

The story problems that follow are provided to illustrate the meanings of multiplication that students will encounter. Vary the contexts and the numbers to ensure that story problems are meaningful to students. Teachers might include the following types of questions.

- Brooke had 5 bags of apples. There are 12 apples in each bag. How many apples does she have in all? (equal groups, multiplication, set model)
- Brandy organized the chairs into 15 rows with 14 chairs in each row. How many chairs in all? (equal groups, multiplication, area model)
- A frog can jump 2 m in one leap. If he takes 37 leaps, how far did he go? (equal groups, multiplication, linear model)
- Nicole has 90 strawberries. She has six bags. She wants to put the same amount of strawberries in each bag. How many strawberries will she put in each bag? (equal groups, partitioning, set model)
- Megan organized the 144 chairs into rows of 6. How many rows were needed? (equal groups, measurement division, array model)
- Kathy put new tiles in her room. She used 30 tiles, each measuring one square metre. If one side of her room is 5 m long, what is the length of the other side? (equal groups, partitioning, area model)
- Brylen has 3 m of ribbon. Serena has 16 times as much ribbon as Brylen. How much ribbon does Serena have? (compare, multiplication, linear model)
- Aurora saved $124 from doing chores. Aurora saved four times as much as Tamara. How much did Tamara save? (compare, initial unknown)
- If five students are going to reading club and 15 students are going to mathematics club, how many times larger is mathematics club than reading club? (compare, multiplier unknown)
- Gavin has 3 pairs of pants and 15 shirts. How many different combinations of a pair of pants and a shirt can he make? (combination, multiplication)
- Gavin has 75 different outfits he can make from his jeans and shirts. If he has three pairs of pants, how many shirts does he have? (combination, division)

It is important for students to both solve and create story problems for simple multiplication situations. This will help to enhance students' understanding of the many contexts in which multiplication and division are used.

N05.07 Students should be able to determine the product of two given numbers using a personal strategy, and they should record the process symbolically. Many different algorithms (symbolic recordings) are possible. Teachers should encourage students to develop and discuss multiple approaches and talk about which strategy works best in various contexts. Often the strategy chosen depends on the numbers involved.

Personal strategies make sense to students and are as valid as the traditional algorithm. Therefore, emphasis should be on students' algorithms rather than on the traditional algorithm. The paper-and-pencil recording of students' personal strategies should reflect their thinking and must be reliable, accurate, and efficient. Most important is that students can justify how and why an algorithm works.

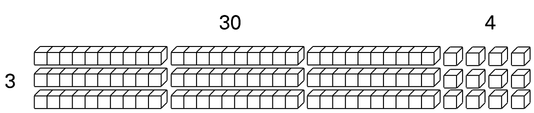
It is important to monitor the types of strategies that students are using. Students should be encouraged to refine their strategies to increase their efficiency. While personal strategies should be encouraged and accepted, when those strategies become inefficient, students should make a transition to more efficient strategies. These more efficient strategies will serve them better as they move to more complex mathematical situations. For example, a student may use repeated addition to solve 6 × 24, (24 + 24+ 24+ 24 + 24 + 24). Although it is a strategy that will result in a correct solution, the strategy is not efficient. Moving to a multiplication strategy is both efficient and effective.

Teachers should monitor each student’s symbolic recording of their strategy to ensure that it is accurate, mathematically correct, organized, and efficient. In particular, teachers should monitor student algorithms to ensure that the equal sign is used correctly. For example, to solve 12 × 15, a student may use a personal strategy and think, 10 groups of 15 is 150. Two groups of 15 is 30. 150 plus 30 is 180. The student would then record this thinking symbolically. For example, the student might incorrectly record their thinking as, 12 × 15 = 10 × 15 = 150 = 2 × 15 = 30 = 150 + 30 = 180. This recording demonstrates an incorrect use of the equal sign. The teacher would need to model how to record the student’s thinking accurately and correctly on paper as shown below.

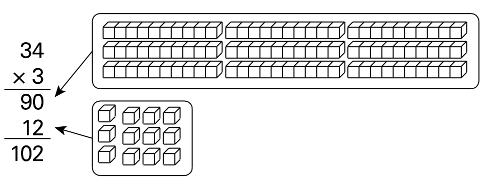
12 × 15 =	or	15
10 × 15 = 150		× 12
2 × 15 = 30		150
150 + 30 = 180		+ 30
		180

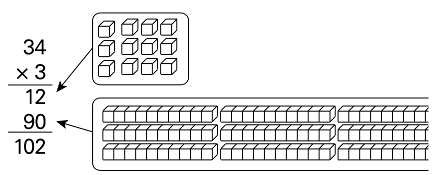
Many algorithms are possible, some of which are shown above and below.

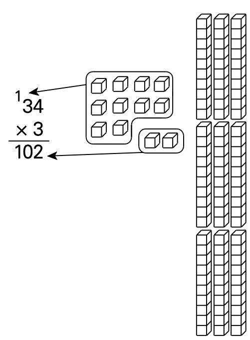
Algorithms for 3 × 34:



3 × 34 =
3 × 30 = 90
3 × 4 = 12
90 + 12 = 102

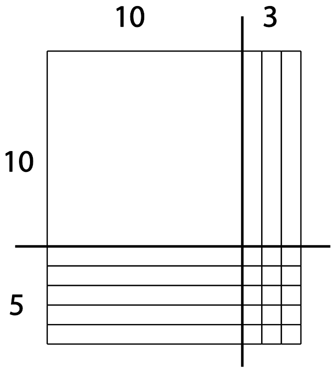






15 × 13 = (10 + 5) × (10 + 3)	13
10 × 10 = 100	× 15
10 × 3 = 30	100
5 × 10 = 50	30
5 × 3 =15	50
100 + 30 + 50 + 15 = 195	+ 15
	195

15 × 13



15 × 13 = 15 × (10 + 3)
15 × 10 = 150
15 × 3 = 45
150 + 45 = 190

13	13
× 15	× 15
130	15
50	50
+ 15	30
195	+ 100
	195

SCO N06 Students will be expected to demonstrate, with and without concrete materials, an understanding of division (three-digit by one-digit), and interpret remainders to solve problems. [C, CN, PS]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N06.01 Model the division of two given numbers, using concrete or visual representations, and record the process symbolically.

N06.02 Explain that the interpretation of a remainder depends on the context.
- Ignore the remainder (e.g., making teams of four from 22 people [five teams, but two people are left over]).
- Round the quotient up (e.g., the number of five-passenger cars required to transport 13 people).
- Express remainders as fractions (e.g., five apples shared by two people).
- Express remainders as decimals (e.g., measurement and money).

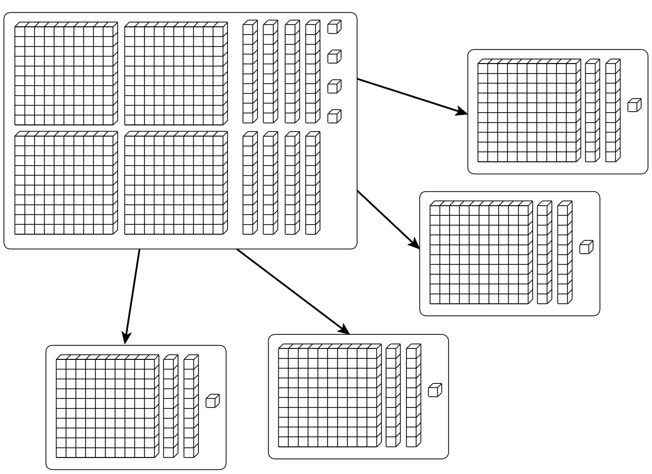
N06.03 Solve a given division problem in context, using personal strategies, and record the process.

N06.04 Create and solve division story problems, and record the process symbolically.

N06.05 Determine the quotient of two given numbers using a personal strategy and record the process symbolically.

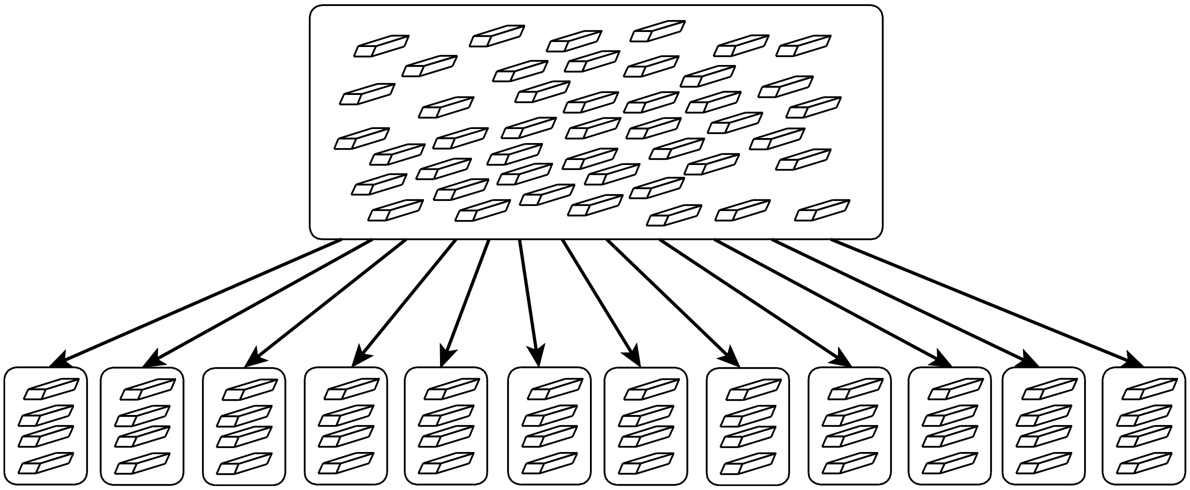
Performance Indicator Background

N06.01 Students should be able to use base-ten blocks to model the solution to a problem requiring them to identify how many in each group, and they should record a picture of their work. For example, There are 484 stickers. They are shared equally by 4 students. How many stickers will each student get?



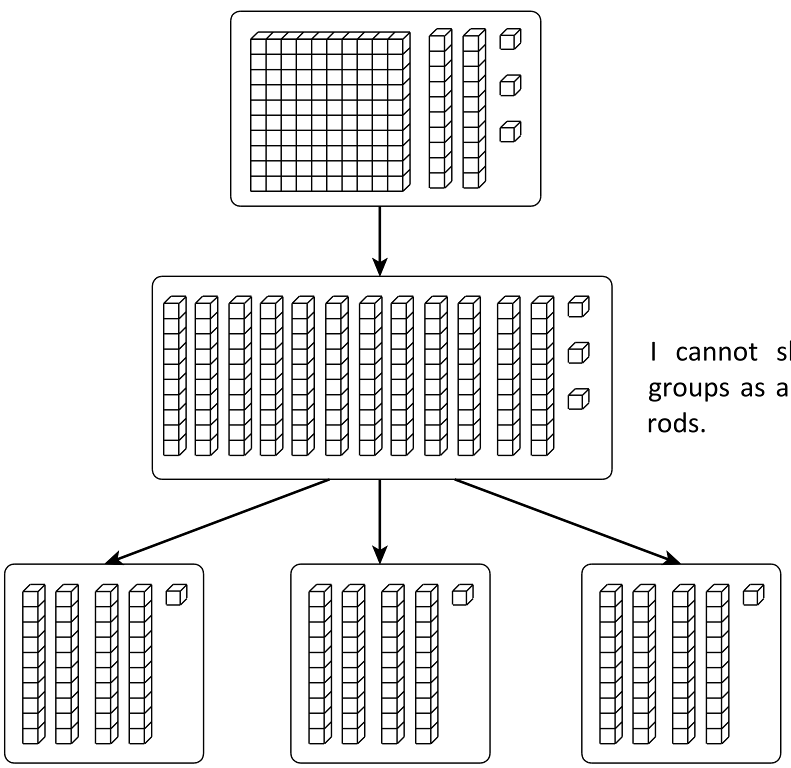
Students should be able to use base-ten blocks to model the solution to a problem requiring them to identify how many groups (repeated subtraction), and then they should record a picture of their work.

There are 48 erasers on the table. I need to put 4 in each container. How many containers will I need?



While students have learned that division can mean fair sharing/partitioning (finding the size of a group) or measurement division (finding the number of groups of equal size), it is often helpful when dividing a larger number by a small number to use the fair-sharing model. For example to divide 123 ÷ 3, it is easier to think of this as sharing 123 in three equal groups, rather than sharing it in groups of three.

I have 123 books. I want to place them on three shelves on a bookcase. How many books will I put on each shelf?



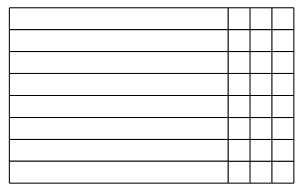
I cannot share the flat with the three groups as a whole flat, so I trade it for 10 rods.

Now I have 12 rods to share, so I can put 4 in each group. I then have 3 little cubes to share and I can put one in each group, so each group has 41.

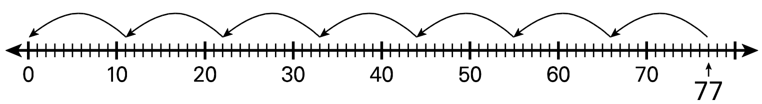
Students can also use area models or arrays to divide, the solution being the missing dimension. This uses division's relationship with multiplication.

There are 104 chairs in the gymnasium. The chairs are arranged in 8 rows. How many chairs will be in each row?

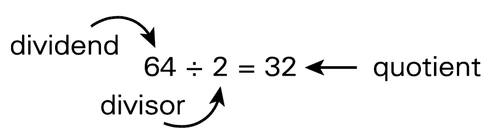
The area is104.
I make 8 rows.
I put 13 in each row.



A kangaroo can travel a distance of 5 metres each time it hops. If a kangaroo hopped a total of 77 metres, how many times did it hop?



N06.02 Providing students with base-ten blocks allows them to solve the problems and discuss the concept of remainders. Teachers can work with students to demonstrate ways of documenting their thinking. Students should understand that the **remainder** (the number of units left over) must be less than the **divisor**. Models help to clarify this idea. Students also need to know that the answer for a division sentence is the **quotient** and the number to be divided is the **dividend**.



Students should understand that when solving division problems, remainders are handled differently depending on the context. They should recognize when a remainder is significant for decision making. For example, the remainder

- needs to be ignored—When you want to know how many $2 notebooks can be bought with $11, the answer if five, since there is not enough money to buy six notebooks.
- needs to be rounded up—When you want to know how many four-passenger cars are needed to transport 27 children, the answer is 7 since you cannot leave anyone behind.
- must be addressed specifically—When 91 students are to be transported in three buses, there may be 30 students on two buses and 31 on the other because you cannot leave anyone behind.
- is best described as a fraction—When four children share nine oranges, each gets two oranges and $\frac{1}{4}$ of the remaining orange.

N06.03 and **N06.04** Students must be able to create and solve division story problems that reflect different meanings for division. These meanings include equal groups, comparison, and combinations.

EQUAL GROUPS

The equal groups structure involves determining the size of a group (partition division) or the number of equal groups (measurement division). The “equal groups” structure may be modelled with sets, arrays (or area models), and linear or measurement models such as number lines.

Equal Groups—Partition Division

- A student may use a set model to determine the size of a group. For example, Bill has 360 apples. He wants to share them equally among his five charities. How many apples will each charity receive?
- A student might use an array model to determine the size of a group. For example, How many chairs should you put in each row if you need to organize 186 chairs into six rows?

Equal Groups—Measurement Division

- A student might use a number line when they need to know how many jumps it will take to cover an 84 m distance, if they can jump 3 m each time.

COMPARISON

Additionally, division is used in comparison situations. Models used for comparison may involve sets, arrays, and linear models as well.

Comparison—Comparing Sets

- A student might be asked to compare sets. For example, the soccer team collected 675 recyclable containers during the recycling drive. That was three times as many containers as the hockey team collected. How many recyclable containers did the hockey team collect? To model this, a student might build a set of 675. Then, they might build another set of 675 and divide the set into three equal groups, taking one of the three groups to show that the hockey team collected 225 recyclable containers.

Comparison—Comparison Rate/Multiplier

- A student might also be asked to determine the comparison rate or multiplier for two given sets or models. For example, if the height of an office building is 140 m and the height of a school is 7 m, how many times as tall as the school is the office building? A student might model this using base-ten materials or a number line.

COMBINATIONS

The third structure is the combinations structure, which provides the foundation for later work in probability. For division, students may be asked to find the size of one set given the product and the other set. For example, the cafeteria says that it can create 108 different meals made up of vegetables and a main dish. The cafeteria serves nine different vegetables. How many main dishes must it have?

N06.05 It is expected that, by the end of the year, students will be able to symbolically divide up to three-digit numbers by a one-digit divisor using reliable, accurate, and efficient strategies. While some of these strategies may have emerged directly from students’ work with base-ten blocks, other strategies should be modelled by students using the base-ten blocks to help understand the logic behind them. Students should be able to explain the strategy used and whether the solution is reasonable based on their prior estimate. Through the sharing of strategies, students will be exposed to a variety of possible division strategies, and each student will adopt ones that he or she understands well and has made his or her own. That is why these strategies are often referred to as “personal strategies.” The most appropriate strategy used may vary depending on the student and the numbers involved in the problem.

Personal strategies make sense to students and are as valid as the traditional algorithm. Therefore, emphasis should be on students’ algorithms rather than on the traditional algorithm. The paper-and-pencil recording of students’ personal strategies should reflect their thinking and must be reliable, accurate, and efficient. Most important is that students can justify how and why an algorithm works. Students should be encouraged to refine their strategies to increase their efficiency, and teachers should monitor each student’s symbolic recording of the strategy to ensure that the recording is accurate, mathematically correct, organized, and efficient.

Examples of personal strategies and their symbolic recordings are shown below.

- I had to solve 363 ÷ 3. I took 3 flats, 6 rods, and 3 small cubes, and I needed to divide them into three groups. So, I put 1 flat in each of the three groups, and that used up 3 flats or 300. I put 2 rods in each of the three groups. So I used up 6 rods or 60. That left me with 3 small cubes. I put 1 small cube in each group. So 363 divided by 3 is 121.

```
3)363
– 300      (100)
   63
 – 60       (20)
    3
  – 3        (1)
    0
```

  363 ÷ 3 = 121

- I had to divide 363 into three groups. I broke 363 up into 300, 60, and 3. I knew that 300 divided into three groups would give me 100 in each group. Then, 60 divided into three groups gave me 20 in each group. Then, I worked with the 3. I knew I could put one in each group. So, I had 100, 20, and 1 in each group. So, 363 divided by 3 is 121.

  363 ÷ 3 = (300 + 60 + 3) + 3
  300 ÷ 3 = 100
  60 ÷ 3 = 20
  3 ÷ 3 = 1

  363 ÷ 3 = 121

- I had to divide 363 into 3 groups. I broke 363 up into 3 hundreds, 6 tens, and 3 ones. I knew that 3 hundreds divided into three groups would give me 1 hundred in each group. I knew that 6 tens divided into three groups would give me 2 tens in each group. Then, I worked with the 3 ones. I knew I could put 1 one in each group. So, I had 2 tens and 1 one in each group. So, 363 divided by 3 is 21.

$$
\begin{array}{r}
121 \\
3\overline{)363} \\
\underline{-300} \\
63 \\
\underline{-60} \\
3 \\
\underline{-3} \\
0
\end{array}
$$

- I had to divide 269 into four groups. I broke 269 up into 200 + 40 + 29. I started with 200 because I knew that four groups of 50 are 200. Then, I knew that four groups of 10 are 40. So, that left me with 29. I knew that 4 × 7 is 28, and I would have 1 left over.

269 ÷ 4 = ?

200 ÷ 4 = 50

40 ÷ 4 = 10

29 ÷ 4 = 7 remainder 1

269 ÷ 4 = 67 R. 1

or

$$
\begin{array}{rl}
4\overline{)269} & \\
\underline{-200} & \underline{(50)} \\
69 & \\
\underline{-40} & \underline{(10)} \\
29 & \\
\underline{-28} & \underline{(7)} \\
1 &
\end{array}
$$

SCO N07 Students will be expected to demonstrate an understanding of fractions by using concrete, pictorial, and symbolic representations to ▪ create sets of equivalent fractions ▪ compare and order fractions with like and unlike denominators [C, CN, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

N07.01 Represent a given fraction of one whole, set, linear model, or region using concrete materials.
N07.02 Create a set of equivalent fractions, and explain, using concrete materials, why there are many equivalent fractions for any given fraction.
N07.03 Model and explain that equivalent fractions represent the same quantity.
N07.04 Determine if two given fractions are equivalent, using concrete materials or pictorial representations.
N07.05 Identify equivalent fractions for a given fraction.
N07.06 Compare and order two given fractions with unlike denominators by creating equivalent fractions.
N07.07 Position a given set of fractions with like and unlike denominators on a number line, and explain strategies used to determine the order.
N07.08 Formulate and verify a personal strategy for developing a set of equivalent fractions.

Performance Indicator Background

N07.01 In order for students to construct a firm foundation for fraction concepts, they need to experience and discuss activities that promote the following understandings:

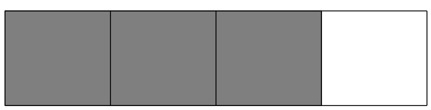
- "Fractional parts are equal shares or equal-sized portions of one whole or unit. A unit can be an object or a collection of things. More abstractly, the unit is counted as 1. On the number line, the distance from 0 to 1 is the unit.
- Fractional parts have special names that tell how many parts of that size are needed to make the whole. For example, *thirds* require three parts to make one whole.
- The more fractional parts used to make one whole, the smaller the parts. For example, eighths are smaller than fifths.
- The denominator of a fraction indicates by what number the whole has been divided in order to produce the type of part under consideration. Thus, the denominator is a divisor. In practical terms, the denominator names the kind of fractional part that is under consideration. The numerator of a fraction counts or tells how many of the fraction parts (or the type indicated by the denominator) are under consideration. Therefore, the numerator is a multiplier—it indicates a multiple of the given fractional part." (Van de Walle and Lovin 2006a, 251).

$$\frac{\text{numerator}}{\text{denominator}} = \frac{\text{parts being considered}}{\text{total number of parts in one whole}}$$

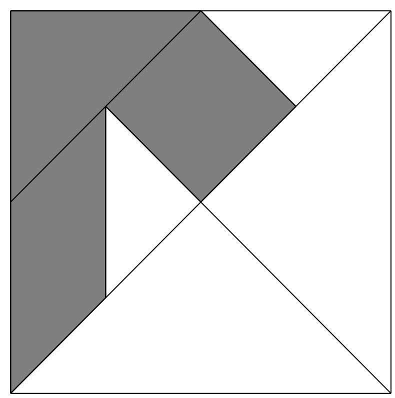
Concrete materials must be used to develop fractional concepts adequately, therefore a variety of materials are effective. Pattern blocks are very useful models. Using pattern blocks as concrete representations for either fractions of one whole or fractions of a set can help students make connections between the two models. For example,

- the triangle is $\frac{1}{3}$ of the trapezoid (fractions of one whole)
- The triangle is $\frac{1}{4}$ of a set of four blocks (made up of a triangle, two squares, and a rhombus) (fractions of a set).

Part of One Whole: This is when one unit is partitioned into equal parts. The sharing of food items or a piece of paper is commonplace to students. The more opportunities they have to partition fairly, the better their visual concept will be for fractions. The emphasis should be on equal parts or fair shares. Students should understand that while the parts are equal in area, they do not need to be identical; this can be a misconception. A tangram set demonstrates this idea clearly where the square, the medium-sized triangle, and the parallelogram all have equivalent area but are not identical in shape. It is important that the representation of the whole, one whole or one, is clear so students understand which region they are taking apart; this concept is essential for comparing fractions. The parallelogram, the square, and the medium-sized triangle each represent $\frac{1}{8}$ of the whole region.

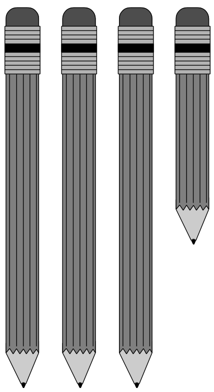


$\frac{3}{4}$ of the bar is shaded.



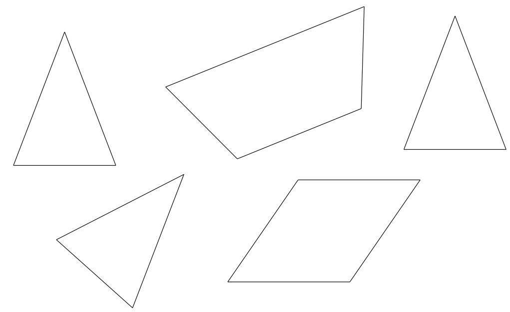
Part of a set: An important point relating to fractions of a set is that the equal parts into which one whole is divided are equal but do not have to be identical. Students may be easily confused by sets that contain different items or are different shapes. For example, in a group of 8 people, there may be 5 children, 2 women, and 1 man. It is still possible to identify the fraction of the set represented by the women as $\frac{2}{8}$, by the children as $\frac{5}{8}$, or the man as $\frac{1}{8}$.

The model below involves finding fair shares of a set of objects. This generally involves partitioning one at a time to each person to ensure that the sharing is fair. This concept of fractions requires students to view the total number as one unit; therefore, initial experiences should be with sets that are contained, such as a box of 10 pencils, a package of 8 erasers, a carton of eggs, etc. This helps to solidify this concept. Students can connect this concept of fractions with division or fair sharing; for example, if 15 books are shared equally with 3 children, each person gets $\frac{1}{3}$ of the original pile of books. In this case, students might not get exactly the same books, but they each get $\frac{1}{3}$ of the set of books. This points to the idea that a group of objects do not need to be identical to be shared.



part of a set

$\frac{3}{4}$ (part of the pencils that are long)



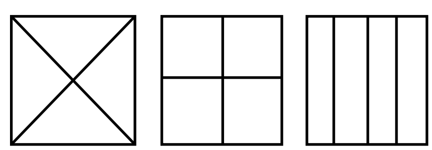
$\frac{3}{5}$ of a set

(part of the shapes that are triangles)

The meaning for part of one whole can be extended to the part of a set meaning in certain situations. For example, when sharing a pizza that has been cut into 8 equal pieces, students can see that one-half also means 4 of the 8 pieces. Also, we can say $\frac{1}{4}$ of the pizza is eaten or $\frac{2}{8}$ of the slices are eaten. Tangrams can also be used for this meaning of fractions: $\frac{5}{7}$ of the Tangram pieces are triangles; $\frac{6}{8}$ area of the whole region is made up of triangles.

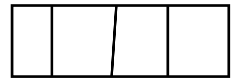
Part of measure: Fractions of measure, at this level, involve the measure of length such as finding a fractional unit on a number line. Fraction strips, Cuisenaire rods, number lines, line segments, and rulers are used for length.

Fractions are students' first experiences in which a number represents something more than a count. Students should investigate the more common fraction families such as halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths. Provide opportunities to explore other fractions in problem situations and in literature. They will need experiences with a variety of materials, including, among others, fraction pieces (Fraction Factory / Cuisenaire rods) geo-boards, counters, coloured tiles, pattern blocks, egg cartons, grid paper, folded papers, and circle pieces. Be flexible using the manipulatives and vary the pieces representing one whole. Provide many and varied opportunities for students to estimate fractional quantities and to explore the idea of simple fractions in meaningful situations. Initial experiences should be with part-of-a-whole situations then making connections to the other fraction models. It is also helpful when examining a situation involving a fraction, to show the related fraction (e.g., if one-third of a pie is eaten, then two-thirds of that pie remains). Informal experiences will help students see that when wholes are divided into a greater number of fair shares, the size of the shares will decrease; this will help later when comparing fractions.



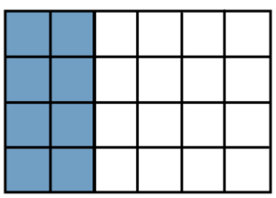
Students should see that there are many ways to make the same fractional part. Using pattern blocks where the hexagon is designated as the whole, students could find how many different ways they can make $\frac{1}{2}$, $\frac{1}{3}$, etc. Or using a square, find how many different ways to make $\frac{1}{4}$. This can help with the understanding of equivalence.

It is important that students see and represent non-examples of the area model for fractions. Each piece does not represent $\frac{1}{4}$ area of the whole.

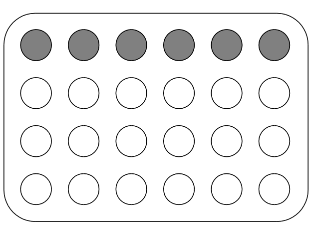


Continue to use language such as “one of three equal parts” and help students connect the language with its symbol $\frac{1}{3}$. Point out to students that $\frac{1}{4}$ may be read either “one-fourth” or “one-quarter.” The money application of four quarters makes a whole dollar can be connected to this use of the word **one-quarter**. To assist with clarity of meaning, always write fractions with a horizontal bar.

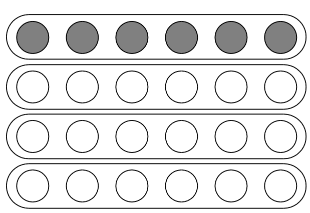
N07.02 and **N07.05** When exploring sets of equivalent fractions, it is important to have students explore creating equivalent fractions that have numerators and denominators that are greater numbers than the original ($\frac{1}{2}=\frac{3}{6}$) as well as those that have numerators and denominators that have lesser values than the original ($\frac{4}{6}=\frac{2}{3}$). This enables teachers to engage students in discussion about the terminology of simplest-term fraction. Students should be exposed to the terminology of **simplest terms** and should not use rules to determine these, but rather use reasoning to decide if a fraction is the simplest fraction that can be used to explain a region. Students should be encouraged to talk about how they know this is the simplest fraction using pictures, symbols, and words. For example, a student might be asked to name the fraction represented by the picture below in a variety of ways and then explain which is the simplest term fraction to describe it.



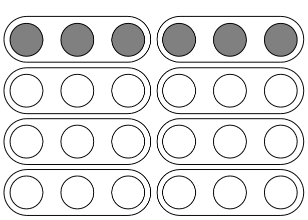
Some manipulative materials that illustrate equivalent fractions include fraction circles, pattern blocks, geo-boards or geo-paper, fraction strips, Fraction Factory, and egg cartons.



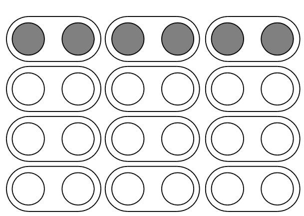
$\frac{6}{24}$



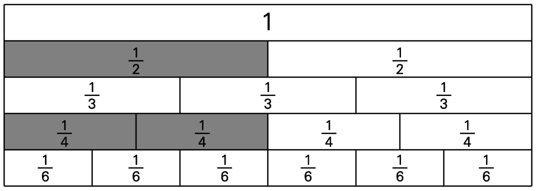
$\frac{1}{4}$



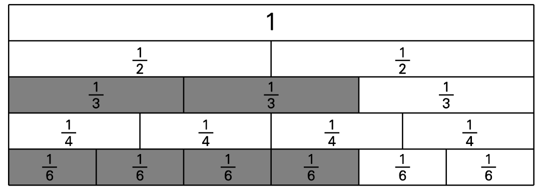
$\frac{2}{8}$



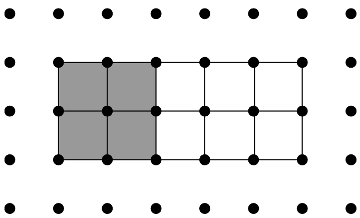
$\frac{3}{12}$



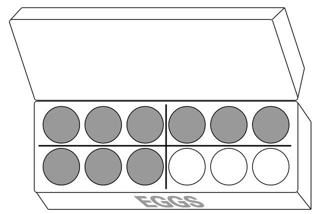
$\frac{1}{2} = \frac{2}{4}$



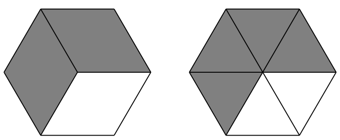
$\frac{2}{3} = \frac{4}{6}$



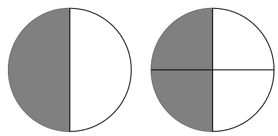
$\frac{2}{5} = \frac{4}{10}$



$\frac{9}{12} = \frac{3}{4}$

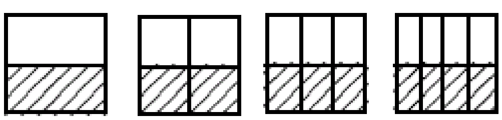


$\frac{2}{3} = \frac{4}{6}$



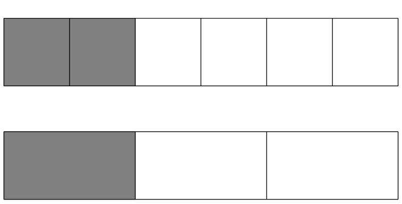
$\frac{1}{2} = \frac{2}{4}$

N07.03 To determine equivalent fractions, students must understand that they are different representations of equal value. To develop a conceptual understanding of equivalency, it is important that models be used to generate the different representations of a fraction. Students must understand why a fraction can have another name (e.g., $\frac{1}{2} = \frac{4}{8}$, and yet have the same value. Students should be able to visualize equivalent fractions as the naming of the same region partitioned in different ways as shown here.



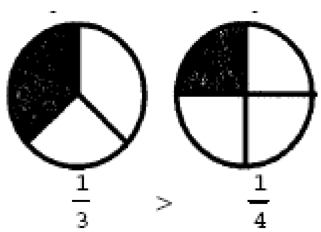
$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$

N07.04 After students have spent time concretely creating fractions that are equivalent, they should be expected to show if two given fractions are equivalent. This can be done using the manipulatives they have previously worked with. Focus students' attention to the concept that the whole regions or whole sets have to be the same size in order to compare them. For example when comparing $\frac{1}{3}$ and $\frac{2}{6}$, the whole must be the same size.



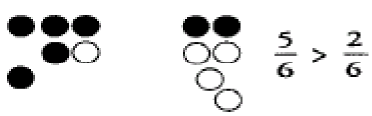
N07.06 Students should understand that they can compare fractions with unlike denominators by renaming one or more of the fractions; that is by creating an equivalent fraction for one or more of the given fractions. For example, when comparing $\frac{1}{2}$ and $\frac{3}{4}$, $\frac{1}{2}$ can be renamed as an equivalent fraction, $\frac{2}{4}$. It is then possible to compare $\frac{2}{4}$ and $\frac{3}{4}$ to determine that $\frac{2}{4} < \frac{3}{4}$ and therefore $\frac{1}{2} < \frac{3}{4}$. To compare $\frac{1}{2}$ and $\frac{2}{3}$, both fractions can be renamed as equivalent fractions. $\frac{1}{2}$ may be renamed as $\frac{3}{6}$ and $\frac{2}{3}$ may be renamed as $\frac{4}{6}$. Students should conclude that $\frac{3}{6} < \frac{4}{6}$ and therefore $\frac{1}{2} < \frac{2}{3}$.

N07.07 To compare and order fractions, students should use three categories of models: area model, length model, and set model. The area model, part of a whole, can be used to compare $\frac{1}{3}$ and $\frac{1}{4}$ as illustrated using fraction circles.



The length model can be used to compare $\frac{1}{2}$ and $\frac{3}{4}$ between 0 and 1 using a number line.

The set model can be used to compare part of a set of like objects such as $\frac{5}{6}$ and $\frac{2}{6}$.



Students may use a variety of strategies for comparing and ordering fractions with like and unlike denominators.

- Students should understand that if two fractions have the same denominator, the size of the numerator will determine which fraction is larger or smaller. For example, $\frac{4}{5} > \frac{2}{5}$ because if a number of equal-sized pizzas are each cut into 5 equal pieces, 2 of those pieces are less than 4 of them.

- Students should also understand that if two fractions have the same numerator, then the one with the larger denominator is less. For example, $\frac{3}{8} < \frac{3}{5}$ because eighths are smaller than fifths.
- Benchmarks (values used for comparison) such as $\frac{1}{2}$ or 1 can be used to compare fractions, (e.g., $\frac{2}{5} < \frac{7}{8}$ because $\frac{2}{5}$ is less than $\frac{1}{2}$, while $\frac{7}{8}$ is more than $\frac{1}{2}$). Students should be encouraged to solve and create problems in context that relate to the comparison of fractions. For example, Erin and Mary each have a piece of rope. Erin cut her rope into eighths and Mary cut hers into twelfths. If Erin kept 6 pieces and Mary 8 pieces, who had the most and how do you find out?
- Rename fractions to make comparison easier.

N07.08 Students should develop, through their work with concrete materials, their personal strategy for determining equivalent fractions, rather than being told to multiply the numerator and denominator by the same amount. They should use reasoning to explain why their strategy for determining equivalent fractions works.

SCO N08 Students will be expected to describe and represent decimals (tenths, hundredths, and thousandths) concretely, pictorially, and symbolically. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N08.01 Write the decimal for a given concrete or pictorial representation of part of a set, part of a region, or of a unit of measure.
N08.02 Represent a given decimal using concrete materials or a pictorial representation.
N08.03 Represent an equivalent tenth, hundredth, or thousandth for a given decimal, using concrete or visual representations.
N08.04 Express a given tenth as an equivalent hundredth and thousandth.
N08.05 Express a given hundredth as an equivalent thousandth.
N08.06 Explain the value of each digit in a given decimal.

Performance Indicator Background

N08.01 and **N08.02** Students understand that decimal notation is an extension of the base-ten system of writing whole numbers and is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on. Decimal numbers represent parts of one whole. Students should relate decimal numbers to reference points (e.g., 0.452 m is a little less than one-half a metre). Students should have used a variety of materials to model and interpret decimal tenths, and hundredths in Mathematics 4. In Mathematics 5, students should extend their understanding of the decimal system to include decimal thousandths.

According to Small (2008, 228–231), students will learn important decimal principles, through the use of concrete materials, pictorial representations, and modelling. Using decimals extends the place-value system to represent parts of one whole. The use of a decimal point must be taught as a symbol that separates the tenths from the ones, or in other words, the “part from the whole.”

- Using decimals extends the place-value system to represent parts of one whole.
- The base-ten place-value system is built on symmetry around the ones place and the decimal.
- Decimals can represent parts of one whole as well as mixed numbers.
- Decimals can be interpreted and read in more than one way. Students should become familiar and comfortable renaming and reading decimals in several ways (e.g., 4.3 may be renamed 43 tenths).
- 5. Decimals can be renamed as other decimals or fractions (e.g., $\frac{600}{1000}$ or 0.600 can be renamed as $\frac{60}{100}$ or 0.60. It can also be renamed as $\frac{6}{10}$ or 0.6; This can be shown pictorially on a thousandths grid.

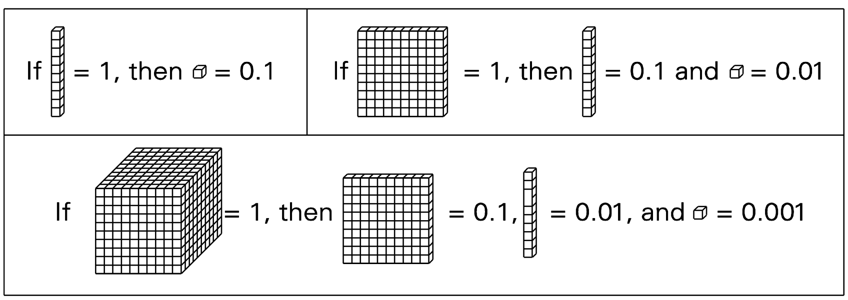
Focus on the need to continue the pattern in our base-ten number system, so that the unit (or one whole) is divided into ten, one hundred or one thousand equal parts (or tenths, hundredths, or thousandths).

Throughout the study of decimals, there are many concrete materials that will aid students in the understanding of decimal concepts including

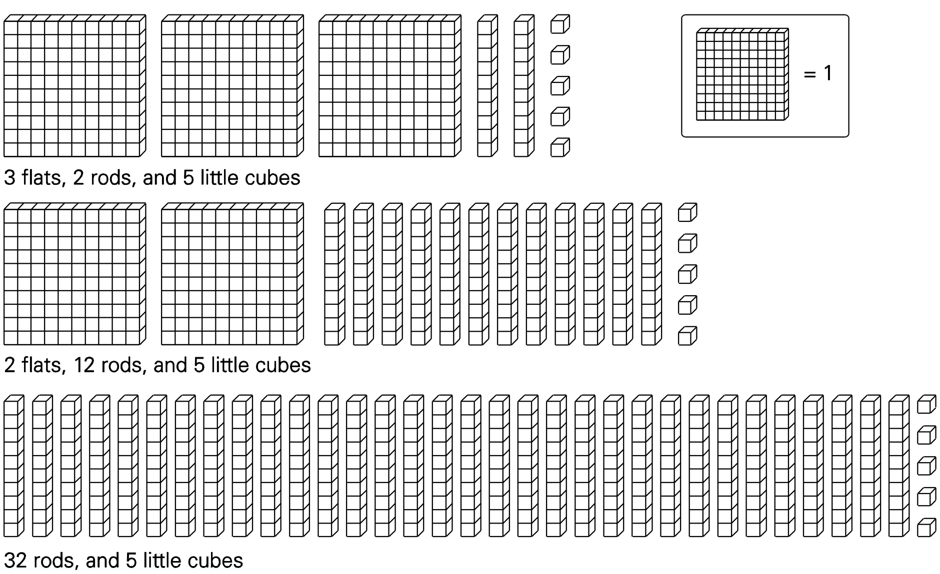
- grid paper (hundredths and thousandths)
- number lines (tenths, hundredths, and thousandths)
- gasoline prices, posted as a tenth of a cent, which is a thousandth of a dollar (e.g., 93.6¢ is $0.936
- metre stick (millimetres are thousandths of a metre and centimetres are hundredths of a metre) decimal squares

Students should read decimal numbers correctly. For example, the decimal 3.2 is read as “3 and 2 tenths” not as “three point two.” Reading 7.23 as 7 and 23 hundredths reveals the important connection between fractions and decimals, but the language “seven point two three” is meaningless.

Students should be encouraged to represent decimal numbers in a variety of ways using concrete and pictorial representations. Base-ten blocks, for example, are great models to use for decimals. Students should understand that some decimals can represent part of one whole (e.g., 0.3 is three-tenths of one whole) or a mixed number (e.g., 2.5 is two and five-tenths). To help students develop the concept of tenths, hundredths, and thousandths, it is essential to clearly establish the one whole that they are, or will be, dividing into ten, one hundred, or one thousand equal parts. As with fractions, flexibility with representing the “one” should be encouraged. Students should also be working with decimals that are more than one, such as 1.235.

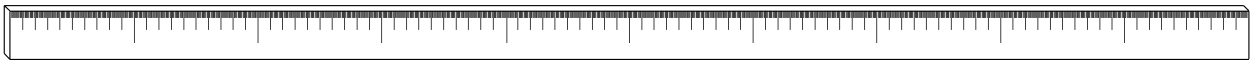


Students should represent decimal numbers in a variety of ways. For example the following diagram shows different ways to represent 3.25. These can be written symbolically as
(3 × 1) + (2 × 0.1) + (5 × 0.01), or (32 × 0.1) + (5 × 0.01), or (2 × 1) + (12 × 0.1) + (5 × 0.01).



All three models show 3.25

A connection can be made to decimals with the work being done with the metric system in outcome M02. Metre sticks can also be used to model decimal numbers. Students can represent tenths, hundredths, and thousandths using length measurements (e.g., 1 mm = 0.001 m, 1 mm = 0.1 cm, 1 cm = 0.01 m), and therefore, 0.423 m can be represented as 423 mm or as 42.3 cm.



N08.03, **N08.04**, and **N08.05** Students should be able to read and interpret decimal numbers in more than one way. For example, if representing two-tenths with base-ten blocks, (with the large cube representing 1), students might use either 2 flats, 20 rods, or 200 small cubes, as all of these have a value of two-tenths of the large cube. Students should also understand that 0.2 may be expressed as an equivalent hundredth (0.20) or thousandth (0.200), and that 0.03 may be expressed as an equivalent thousandth (0.030).

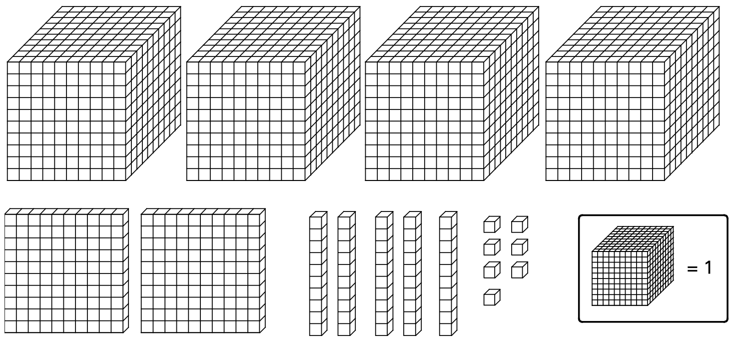
Students should be able to read decimal numbers in print and record the numeric form of decimals upon
- hearing them orally
- seeing them written out in words
- when presented with concrete or pictorial models

When reading numbers, the word **and** is reserved for the decimal. For example, 5.321 is read as five and three hundred twenty-one thousandths, not as five point three two one, or five decimal three hundred twenty-one. Students should also have experience reading numbers in several ways. For example, 6.83 may be read as 6 and 83 hundredths, but might also be read as 6 and 8 tenths, 3 hundredths, or as 6 tenths, 3 hundredths. 1.308 may be read as 1 and 3 tenths, 8 thousandths; as 1 and 30 hundredths, 8 thousandths, or as 1 and three hundred eight thousandths.

Teachers will model the correct reading of whole numbers and decimal numbers and will use the word and only for the decimal point (e.g., 16.8 is read as sixteen and eight-tenths), while 1235 is read as one thousand two hundred thirty-five. It is also important to acknowledge that in everyday use people often read numbers in ways that are not mathematically accurate, such as reading 0.34 as zero decimal thirty-four or zero point thirty-four.

N08.06 Students should recognize that decimals extend the place-value system. Students should use prior knowledge of the patterns associated with place value to explain how tenths, hundredths, and thousandths fit into the place-value system. Students should know that the first place to the right of the decimal point is tenths because ten of these would make one whole (the place to the left of it). The second decimal place is hundredths because it would take ten of these to make one tenth, and ten-tenths to make the whole, thus one hundred of these will make one whole. Similarly, students should learn that the third place to the right of the decimal point is thousandths because it would take ten of these to make one hundredth, and it takes ten hundredths to make one tenth, and ten tenths to make one whole, thus one thousand of these would make one whole.

Students should represent decimals using concrete materials such as base-ten materials and use these models to demonstrate the place value of decimals. The value of a decimal number is the product of the face value of the digit and its place value in the base-ten system
(e.g., 4.257 = (4 × 1) + (2 × 0.1) + (5 × 0.01) + (7 × 0.001). This can be represented with base-ten material as shown below.



Students should recognize and work with the idea that the value of a digit varies depending on its position or place in a numeral. Students should recognize the value represented by each digit in a number as well as what the number means as a whole. The digit "2" in 2.3 represents 2 ones whereas the digit "2" in 3.2 represents 2 tenths; the digit "2" in 3.02 represents 2 hundredths; and the digit "2" in 3.042 represents 2 thousandths. Students should be able to explain the meaning of the digits, including numerals with all digits the same (e.g., for the numeral 2.222, the first digit represents 2 ones the second digit 2 tenths, the third digit 2 hundredths, and the fourth digit 2 thousandths).

It is important to spend time developing a good understanding of the meaning and use of zero in numbers. Students need many experiences using base-ten materials to model numbers with zeros as digits. Teachers should ask students to write the numerals for numbers such as seven and five-hundredths, ninety and two-tenths, or zero and five-hundredths. When writing a number, such as seven and five-thousandths in its symbolic form using digits (7.005), the 0 digits are place holders. If the 0 digits were not used, the number would be recorded as 7.5, and you would mistakenly think that the 5 represented five-tenths instead of five-thousandths. Students need many experiences using base-ten materials to make connections with the symbols for numbers with zeros as digits.

SCO N09 Students will be expected to relate decimals to fractions and fractions to decimals (to thousandths). [CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N09.01 Express, orally and symbolically, a given fraction with a denominator of 10, 100, or 1000 as a decimal.

N09.02 Read decimals as fractions (e.g., 0.45 is read as zero and forty-five hundredths).

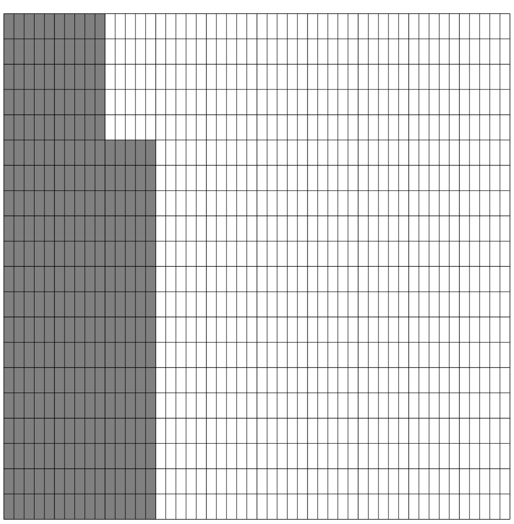
N09.03 Express, orally and symbolically, a given decimal in fraction form.

N09.04 Represent the fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$ as decimals using base-ten blocks, grids, and number lines.

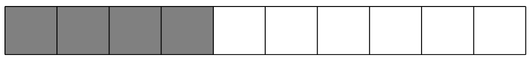
N09.05 Express a given pictorial or concrete representation as a fraction or decimal (e.g., 250 shaded squares on a thousandth grid can be expressed as 0.250 or $\frac{250}{1000}$).

Performance Indicator Background

N09.01, **N09.02**, **N09.03**, and **N09.05** There are different, but, equivalent representations for a number. Relating fractions to decimals is an example of this concept. Connecting fractions to decimals began with the tenths and hundredths in Mathematics 4. Thus, students have had experience with using hundredth grids and base-ten blocks to concretely and pictorially connect fractions to decimals. In Mathematics 5, students should begin by revisiting strategies for determining the exact decimal representation of a fraction with 10 or 100 as the denominator. Then, fractions with 1000 as the denominator can be explored with models. Students should continue to make these connections using models that include base-ten blocks, hundredth and thousandth grids, metre sticks, geo-boards, number lines, hundredth circles, and money. For example, if the thousandths square below represents one whole, then the model below shows two hundred seventy-five thousandths, which could be expressed as 0.275 or $\frac{275}{1000}$, both of which are read as two hundred seventy-five thousandths.



Reading decimals and fractions correctly reinforces the relationship between decimal and fraction tenths, hundredths, and thousandths. Students will see that the model shown below can be read as "four-tenths" which they should connect with two different symbolic representations: $\frac{4}{10}$ or 0.4.



By exploring representations such as these, students should be able to recognize that fractions with a denominator of 10 can be written as decimals as well. Students should also use models, such as base-ten materials, and hundredths and thousandths grids to model decimals and fractions with denominators of 100 and 1000. Students should use these models to explain why the numbers represented can be expressed as a fraction with a denominator that is a power of ten or as a decimal.

Reinforce the connection between decimals and fractions by having students write the fraction and the decimal represented by a base-ten display. Conversely, provide students with decimals or fractions and have them shade the appropriate amounts on the hundredth and/or thousandths grids. Encourage them to write the decimal and fraction for the unshaded part and compare the numbers they wrote for the shaded and unshaded parts. For example, if 0.425 is shaded, then 0.575 is unshaded. They should see that the sum of the shaded and unshaded parts is 1.

N09.04 Students can make use of their knowledge of equivalent fractions (outcome N07) to determine the exact decimal representation for simple fractions ($\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$) that can easily be changed to a fraction that has a denominator that is a power of ten. Students may use a hundredth grid to show that one-fourth is represented by 25 of the 100 squares to show that $\frac{1}{4} = 0.25$. Students should be able to explain that this works because an equivalent fraction for $\frac{1}{4}$ is $\frac{25}{100}$ and that $\frac{1}{4} = \frac{25}{100}$. Similarly, students can use a ten-frame to show that $\frac{1}{2}$ is equivalent to five tenths; a hundredth grid to show that $\frac{1}{2}$ is equivalent to fifty hundredths, or a thousandth grid to show that $\frac{1}{2}$ is equivalent to five hundred thousandths. Students should also explain that $\frac{1}{2} = 0.5$; $\frac{1}{2} = 0.50$ and $\frac{1}{2} = 0.500$. Number lines and base-ten blocks may also be used to show these equivalent decimals and fractions. Students should be encouraged to explain the reasoning they used to determine the equivalent decimal representations using pictures, symbols, and words.

SCO N10 Students will be expected to compare and order decimals (to thousandths) by using benchmarks, place value, and equivalent decimals. [CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N10.01 Compare and order a given set of decimals by placing them on a number line that contains the benchmarks 0.0, 0.5, and 1.0.
N10.02 Compare and order a given set of decimals including only tenths using place value.
N10.03 Compare and order a given set of decimals including only hundredths using place value.
N10.04 Compare and order a given set of decimals including only thousandths using place value.
N10.05 Explain what is the same and what is different about 0.2, 0.20, and 0.200.
N10.06 Compare and order a given set of decimals, including tenths, hundredths, and thousandths, using equivalent decimals).

Performance Indicator Background

N10.01, **N10.02**, **N10.03**, **N10.04**, and **N10.06** Students continue to compare or order decimal numbers and extend this to any decimal number up to thousandths. In Mathematics 4, students developed several key ideas when comparing decimals.

- The whole number part of a number is a critical part for comparison. For example, when comparing 2.39 and 4.2, 2.39 < 4.2 because the whole number 2 is less than the whole number 4.
- When numbers have the same whole, the decimal part becomes critical for comparison. For example, when comparing 4.3 and 4.7, both numbers have 4 ones. However, 3 tenths is less than 7 tenths; therefore, 4.3 < 4.7
- When comparing whole numbers, the number of digits provides a sense of the relative size of numbers; that is, a three-digit whole number is always greater than a two-digit whole number. This is not the case with decimals. When comparing decimals, the number of digits is irrelevant; it is the place value of the digits that matters. For example, when comparing the numbers 40 and 12.2, it is incorrect to conclude that 12.2 is a three-digit number and so must be greater than 40 which is a two-digit number.
- It is important to examine the place value of the digits, not just the size of digits. For example, 6.2 < 40 even though both numbers have two digits, and each of the digits of 6.2 is greater than the each of the digits in 40.

When comparing two decimal numbers, it is useful to use manipulative material (e.g., if the base-ten rod represents 1, then 4 small cubes would show 0.4 and 6 small cubes would show 0.6; therefore 0.4 < 0.6. The symbols <, >, and = are used to compare decimal numbers (e.g., 3.42 < 5.63, 4.2 > 3.8, 2.132 = 2 × 1 + 1 × 0.1 + 3 × 0.01 + 2 × 0.001).

Students may use place-value arguments to compare numbers. For example, 0.2 > 0.02 because 0.2 has two-tenths and 0.02 has zero-tenths; or 0.011 < 0.02 because 0.011 has one-hundredth and 0.02 has two-hundredths. Students may also use benchmarks to compare numbers. As with fractions, the first benchmarks that should be developed are 0, 0.5, 1, 1.5, etc. For example, 0.47 < 0.675 because 0.47 < 0.5 and 0.675 is greater than 0.5.

Students can rename fraction benchmarks as decimal numbers (e.g., $\frac{1}{4}$ = 0.25, $\frac{1}{2}$ = 0.5, $\frac{3}{4}$ = 0.75). These benchmarks are easy to locate on a number line, fraction strips, or hundredth grid. When comparing and ordering decimal numbers, students should also use benchmarks such as one-fourth, one-half, and three-fourths (e.g., they can quickly conclude that 0.8 > 0.423 because the former is much more than one-half and the latter is less than one-half).

Students should be able to order lists of decimal numbers. For example, students can order the following from least to greatest and draw segments using these numbers as measurements in centimetres: 7.2, 5.9, 8.3, 4.5, 9.7, and 6.0. They can use four digits to create as many decimals with two decimal places as they can (e.g., when the digits are 3456, the resulting decimals will be 34.56, 34.65, 43.56, 43.65, 65.43, 65.34, 56.34, 56.43, 35.65, and 35.56). Students are asked to place these numbers from least to greatest. They could also vary this by making numbers with four decimal places and again repeat the ordering.

Students should be able to identify the approximate position of decimal numbers on a number line and use these positions when putting a list of decimal numbers in order. For example, given a segment with end points labelled 2 and 4, students should be able to place the following numbers in their approximate locations: 2.3, 2.51, 2.999, 3.01, 3.75, 3.409, and 3.490. Students should also be able to identify decimal numbers that fall between any two given decimal numbers (e.g., 0.15, 0.139, 0.11, and 0.125 are between 0.1 and 0.2). On a number line, students can label the endpoints 0.1 and 0.2. By dividing ten equal intervals between 0.1 and 0.2, they can show hundredths. Students can continue to divide the hundredths to show thousandths. By exploration, students should begin to understand that between any two decimals there are infinitely many decimals.

N10.05 The numbers 0.2, 0.20, and 0.200 are the same because they represent the same amount, that is $\frac{2}{10}$ is equivalent to $\frac{20}{100}$ which is also equivalent to $\frac{200}{1000}$. They are the same because they all contain the digits 0 and 2. They are all decimal numbers. They are different in their representation and in the number of digits used. Also, 0.2 means 2 parts out of 10, while 0.20 means 20 parts out of 100, and 0.200 means 200 parts out of 1000.

SCO N11 Students will be expected to demonstrate an understanding of addition and subtraction of decimals (limited to thousandths). [C, CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

N11.01 Predict sums and differences of decimals using estimation strategies.

N11.02 Use estimation to correct errors of decimal point placements in sums and differences without using paper and pencil.

N11.03 Explain why keeping track of place-value positions is important when adding and subtracting decimals.

N11.04 Solve problems that involve addition and subtraction of decimals, limited to thousandths, using personal strategies.

Performance Indicator Background

N11.01 and **N11.02** Estimation activities should be integrated throughout the study of decimal numbers. Students should make reasonable estimates of sums and differences of decimal numbers and use techniques for rounding. Students can round each decimal number to the nearest whole number (e.g., 9.7 + 3.2, round the 9.7 to 10, and the 3.2 to 3 giving an estimated sum of 10 + 3 = 13). They round each decimal number to the nearest tenth or hundredth (e.g., 2.834 – 1.562, round the 2.834 to tenths to get 2.8, and round the 1.562 to 1.6, giving an estimated difference of 2.8 – 1.6 = 1.2).

To be efficient when estimating sums and differences mentally, students must be able to access a strategy quickly, and they need a variety from which to choose. Some strategies include

- Using referents (e.g., 2.7 + 12.8 might be described as being more than 15 (2.5 + 12.5), but less than 16 (3 + 13); subtracting 12.6 from 20.7 would give an answer between 7.5 (20 – 12.5) and 8.5 (21 –12.5)). Rounding (e.g., 43.5 + 5.2 is approximately 44 + 5).
- Front-end and front-end adjusted addition can be used to add 32.2 + 24.5 + 14.1. A student might add the tens (30 + 20 + 10 is 60), and the ones and tens can be clustered together to make an estimate of ten for a total of 70. Front-end subtraction can be used for 24.9 – 12.7. Students may subtract the tens followed by the ones and ignore the tenths for an estimate of 12.
- Clustering near compatibles (e.g., 46.32 + 134.05 + 68.17 + 55.2, the 46 and 55 together make approximately 100 and the 134 and 68 make approximately 200 for a total of 300. The tenths and the hundredths would be ignored).

N11.03 In Mathematics 4, students added and subtracted decimal tenths and hundredths and are now expected to work with decimal thousandths. It is recommended that students revisit adding and subtracting to hundredths before moving forward to examples involving thousandths. Each principle and algorithm related to whole number operations continues to apply. There are virtually no changes to the explanations for the algorithms when dealing with addition and subtraction of decimals rather than whole numbers.

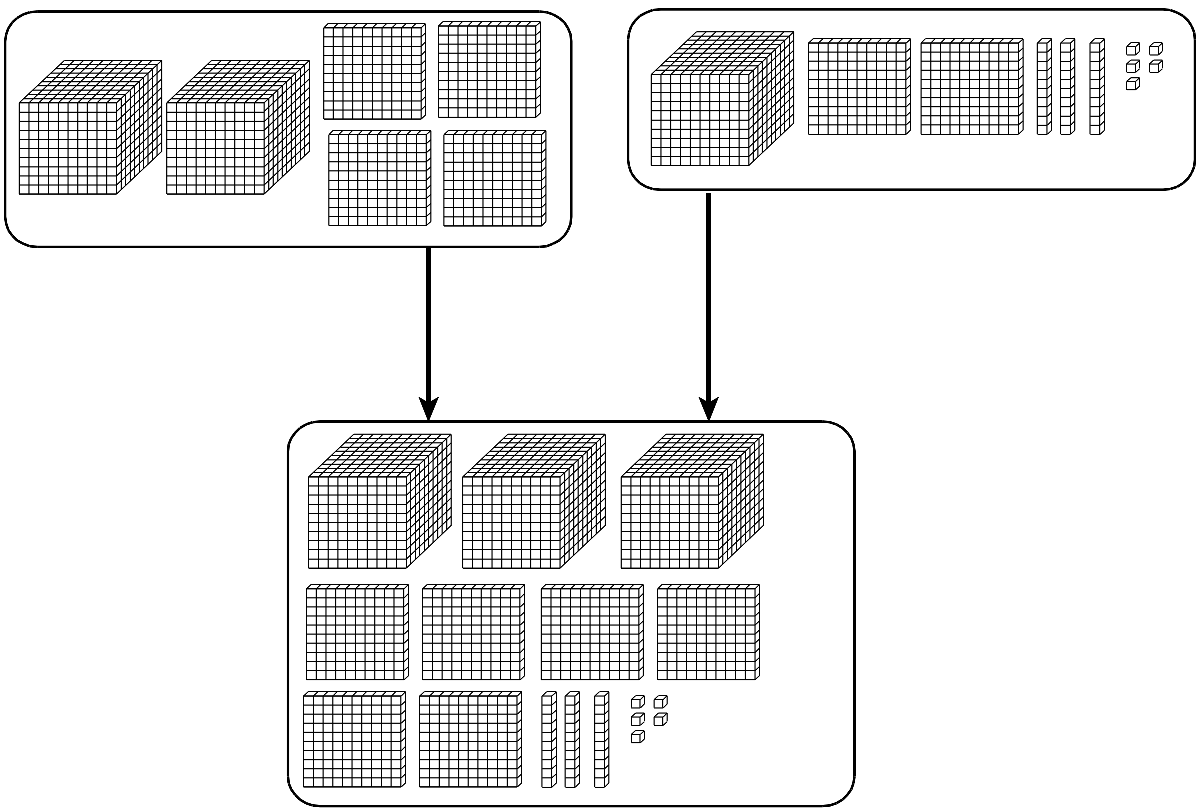
Students should recognize that adding and subtracting decimal numbers is analogous to adding or subtracting quantities of other items (e.g., 3 tenths and 4 tenths are 7 tenths is the same as 3 apples and 4 apples are 7 apples; the same is true with hundredths and thousandths). Students should visualize what each digit represents in base-ten blocks and which blocks would be combined or separated.

Students should develop some computational fluency with decimal numbers. In the past, decimal computation was dominated by lining up the decimal places. While this is important, for accurate computation a firm understanding of place value is needed. Rather than having students line up decimals vertically or add zeros, they should be focusing on place value of the digits.

Before students are introduced to regrouping in addition of decimals, they need to know how to add without regrouping. Smaller decimal numbers should be used as a starting point for introducing addition of decimal numbers.

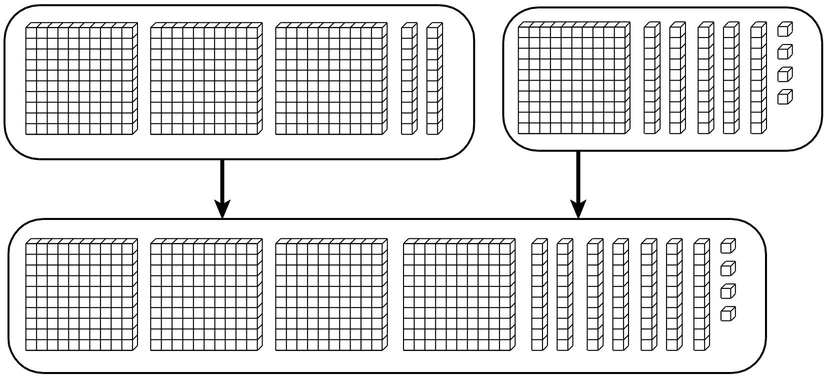
To determine solutions to questions such as 1.62 + 0.3, students might think, I have 1 whole,
6 + 3 tenths, and 2 hundredths, which is 1.92. To subtract 1.4 from 3, they might think, "I have
3 wholes – 1 whole which is 2 wholes. If I subtract 4 tenths from 2 the answer is 1 and 6 tenths, or 1.6.
Alternately, they might think, 3 wholes is the same as 30 tenths, and 1 and 4 tenths is the same as 14 tenths.
30 tenths – 14 tenths = 16 tenths, or 1.6.

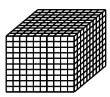
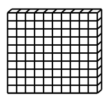
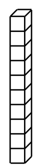
Base-ten blocks and hundredths grids are useful models for students to visualize the addition or subtraction of decimals. When using base-ten blocks, if the large cube is used to represent one whole, 2.4 + 1.235 is modelled as shown below.

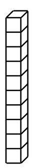


Combining like base-ten blocks provides the sum 3.635.

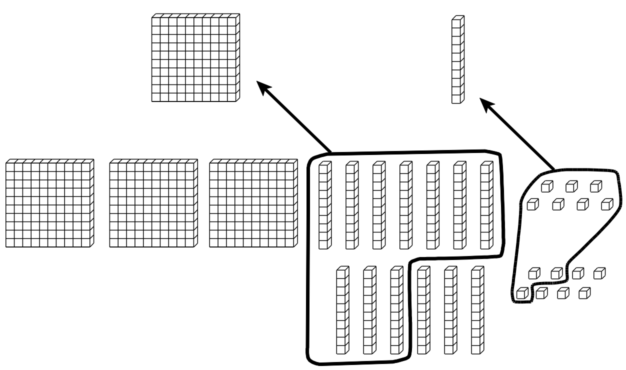
If a flat represents one whole unit, then 3.2 + 1.54 would be modelled as shown below.

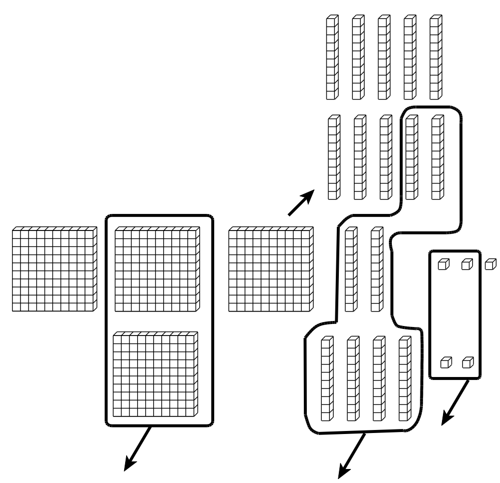


If  = 1, then  = 0.1,  = 0.01, and  = 0.001.

If  = 1, then  = 0.1.

Students should record the process symbolically to make connections with the paper-and-pencil algorithms. Other examples are as follows:





  3.77
+ 0.68
  4.45

Students are encouraged to invent and use personal strategies and algorithms to perform addition and subtraction calculations. Personal algorithms can be instructive for solving addition and subtraction problems as the student develops a better understanding of place value and why these methods work. Personal algorithms can be more conveniently applied in certain situations of addition and subtraction. An algorithm that a student creates is more meaningful to that student and his or her understanding. The more algorithms a student knows, the more efficiently he or she can choose an appropriate algorithm to solve the problem accurately. The application of various algorithms is useful in mental computations. One personal algorithm is to determine partial sums of place values followed by the total sum of the different place values. For example,

  26.1
+ 13.8
  30.0 (add the tens)
   9.0 (add the ones)
   0.9 (add the tenths)
  39.9 (total sum)

One personal algorithm is to subtract in parts, which may make the subtraction easier. To solve 52.4 – 30.2, begin with 52.4, subtract 30, and then subtract two-tenths. This could be recorded as
52.4 – 30 – 0.2 =
52.4 – 30 = 22.4
22.4 – 0.2 = 22.2.

Addition and subtraction questions should be presented both horizontally and vertically to encourage alternative computational strategies. For example to solve 1.234 + 1.99, students might calculate 1.234 + 2 = 3.234 and then 3.234 – 0.01 = 3.224.

The mathematical properties—commutative (order), associative (grouping), zero, and one—have already been explored in earlier grades and with different sets of numbers. However they should be revisited so that students can see their relevance to the addition and subtraction of decimal numbers. Instruction should focus on the usefulness of these properties rather than recognition of names or matching exercises. Discussion should include why certain properties do not apply to subtraction.

N11.04 Students have had experience with a variety of story problem structures working with whole numbers in previous grades. When they learn how to operate with decimals, they should also be exposed to a variety of story problem structures so that they get a full picture of the various contexts in which decimals are used.

Join story problems all have an action that causes an increase. They involve three quantities: the initial amount, the change amount, and the resulting amount.

Separate story problems have an action that causes a decrease. These problems have the same three quantities as join problems: the result amount, the change amount, and the initial amount.

Unlike join and separate story problems, **Part-Part-Whole** story problems do not involve an action. Two parts make up a whole and there is no meaningful difference between the two parts; therefore, there are only two types of part-part-whole questions—whole unknown and part unknown.

Compare problems involve relationships between quantities rather than actions. The third quantity does not actually exist, but is the difference between the two amounts. There are three types of compare problems: difference unknown, larger unknown, and smaller unknown.

Students should be presented with a variety of problems involving each of the structures with different unknowns and should also be asked to create story problems for different representations of these structures.

Students should be solving multi-step story problems involving combinations of the four operations as well as creating their own. Requiring students to create their own problems provides opportunities for them to explore the operations in depth. It is a complex skill requiring solid conceptual understandings and must be part of the student's problem-solving experiences. It is important that, among the problems presented or created by students, some lend themselves to mental computation, others require paper-and-pencil computation, and still others call for the use of calculators. Calculators are useful for decimal computations involving complicated numbers, multiple calculations, and problem solving.

Patterns and Relations

SCO PR01 Students will be expected to determine the pattern rule to make predictions about subsequent terms. [C, CN, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

PR01.01 Extend a given increasing or decreasing pattern, with and without concrete materials, and explain how each term differs from the preceding one.
PR01.02 Describe, orally or in written form, a given pattern using mathematical language such as **one more**, **one less**, or **five more**.
PR01.03 Write a mathematical expression to represent a given pattern, such as $r + 1$, $r - 1$, $r + 5$.
PR01.04 Describe the relationship in a given table or chart using a mathematical expression.
PR01.05 Determine and explain why a given number is or is not the next term in a pattern.
PR01.06 Predict subsequent terms in a given pattern.
PR01.07 Solve a given problem by using a pattern rule to determine subsequent terms.
PR01.08 Represent a given pattern visually to verify predictions.

Performance Indicator Background

PR01.01 Students have had extensive work with understanding increasing and decreasing patterns concretely, pictorially, and symbolically in the previous grades. In Mathematics 5, students will continue to work with increasing and decreasing patterns; however, the focus will be on making and verifying predictions of missing terms/elements in various patterns. The students will use concrete materials and tables to determine pattern rules that will enable them to predict missing terms/elements in a pattern. Given a number, geometric, pictorial, or situational pattern, students should be able to explain the pattern in spoken and written language. Very often, students will need to extend the pattern to fully understand it. Whenever working with patterns and relationships, it is important to use meaningful contexts and use a variety of representations.

Writing a pattern rule for an increasing pattern should describe how each and every term of the pattern is generated and should include the starting point. For example, for the sequence 3, 6, 12, 24, ..., the pattern rule should be “start at three, multiply each term by two.” This is a recursive pattern because each term can be found by applying the same pattern rule to the previous term. The same rule is repeated again and again in the same sequence. At this grade level the patterns should include one operation and the pattern rule should be complete enough so that the rule could be used to find missing or subsequent terms in the pattern. However, this is not very efficient if we need to find the 50th term. Similarly, given the pattern rule, the student should be able to create the pattern from the rule. For example, given the pattern rule “start at two and add 3 to each term,” students should produce the following:
2 + 3 = 5
5 + 3 = 8
8 + 3 = 11
11 + 3 = 14

From these, students will develop the sequence 2, 5, 8, 11, 14, ...

In describing the pattern in the table below, some students may observe that the *A* values are increasing by one (each time). This is recursive thinking, and tells how to find the value of a term given the value of the preceding term. Other students may see that they can determine the value of *A* by adding three to the value of *N*. This is functional thinking focused on the relationship between the term and the term value.

(Term)	*N*	1	2	3	4	5
(Term Value)	*A*	4	5	6	7	8

Input/output or function machines are a good model to use when working with patterns. When any two of the three components (input/output/rule) are known, the third component can be determined. Data from the input/output machine can be translated into a table with the inputs in one column and the outputs in another column. Once a table is created, the students should explore, using guess and check at this time, to find the relationship or pattern rule between the input (position in pattern) and the output (value of number in pattern). The pattern rule can be described using words or symbols (open frames only). Students are now thinking about a relationship between the term and the term value and not a recursive pattern. Each pair of numbers (input, output), forms an ordered pair that can then be graphed on a coordinate grid. Students should be able to describe verbally (oral and written), make connections, and move freely among all the representations. Problems can be created and solved using the different representations.

PR01.02 Through teacher questioning, students should have ample opportunity to explain orally prior to writing descriptions of how elements in various patterns change as the patterns are extended. Have students use manipulatives to copy and extend patterns. Ask them to describe how the concrete representation illustrates the pattern. When describing a pattern, students should be encouraged to state at what number the pattern started and how the number changed.

PR01.03 In Mathematics 4, students used symbols in expressions such as 4 + Δ whereas in Mathematics 5 lower-case letters will be used for the variable. It will be necessary for students to see how 4 + Δ = 7 means the same as 4 + *n* = 7. The symbol Δ has now been replaced by *n*.

This will be students' first formal exposure to the use of variables to represent a number. A variable is a letter or symbol used to represent an "unknown." Patterns using symbols and variables provide a means of describing change mathematically, for example, 2 more than or 6 less than.

When writing the variable expression for a pattern, it is best to limit the study to those patterns where the difference in terms is 1 such as 6, 7, 8, ... or 17, 16, 15, ... If *n* represents the term number in a pattern, then *n* + 5 can be used to describe the pattern 6, 7, 8, ...That is, term 1 is 1 + 5 = 6, term 2 is 2 + 5 = 7 and term 3 is 3 + 5 = 8. To predict the tenth number in the pattern, we can simply write 10 + 5 = 15. That is the tenth number in the pattern 6, 7, 8, ..., 15.

A number sentence is called **an equation**. A number sentence with a variable is an **algebraic equation**. The major difference between an equation and an expression is that an equation is a complete sentence and therefore, contains a verb. For example, *p* = 3 reads "*p* is equal to 3," whereas *p* + 3 reads "*p* plus three." *p* + 3 contains no verb and is, therefore, considered an expression.

In Mathematics 5, variables are, typically, quantities that change. Students might relate variables to things that change over time that are part of their own experiences, such as their height or hair length.

In the early stages of variable usage, it may be wise to avoid the use of “x” as a variable, since students often get “x” mixed up with the multiplication symbol. It is important, when reading aloud to students, to read expressions such as $m + 3$ as “a number *m* added to 3,” or “3 more than *m*.”

PR01.04 Tables are often used to enable students to determine the pattern rule. Tables are used to record the numeric components of patterns such as the number of blocks used for each step. By using a table, students can see the relationship between the terms as well as between the position of the term and its value.

PR01.05 The challenge with patterning or sequencing of numbers is not only to find and extend the pattern, but for students to be able to determine if an element is or is not the next one in the pattern. It is important for students to identify errors in patterns to prevent students from continuing to extend a pattern incorrectly. It is helpful for students to think of a pattern rule and apply it when analyzing tables or charts for errors.

PR01.06 A table or “T-chart” can be constructed to represent a pattern. Once a table is used for the pattern, students may realize that they can extend and predict the pattern without using concrete materials. For those students who find it beneficial to use manipulatives, they should be readily available for their use. The challenge is to predict subsequent elements in a given pattern without actually completing all the entries of the table. When students recognize the pattern, they will be able to determine the fifth, tenth, or even the twentieth element without recording all the elements in between.

PR01.07 Patterns are used repeatedly as a means of developing concepts and as a tool for solving problems. Many problems solved through the use of patterns are appropriate for Mathematics 5 students. For example, use the following pattern to determine the product of 9×999:
2×999
3×999
4×999

PR01.08 This is an opportunity for students to use concrete materials. Students should be able to record the pattern in a given table. When using geometric patterns, students might be asked to describe how to build the next model or other models in the series. This helps to connect the pattern with the model. Students should be asked to represent a pattern using a picture or a number of concrete materials such as pattern blocks or square tiles when appropriate. Enabling students to verify their predictions of elements further in the pattern often requires visual representations.

SCO PR02 Students will be expected to solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions. [C, CN, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

PR02.01 Explain the purpose of the letter variable in a given addition, subtraction, multiplication, or division equation with one unknown (e.g., $36 \div n = 6$).
PR02.02 Express a given pictorial or concrete representation of an equation in symbolic form.
PR02.03 Express a given problem as an equation where the unknown is represented by a letter variable.
PR02.04 Create a problem for a given equation with one unknown.
PR02.05 Solve a given single-variable equation with the unknown in any of the terms (e.g., $n + 2 = 5$, $4 + a = 7$, $6 = r - 2$, $10 = 2c$, $15 \div r = 3$).
PR02.06 Identify the unknown in a problem; represent the problem with an equation; and solve the problem concretely, pictorially, or symbolically.

Performance Indicator Background

Note: The difference between an expression and an equation was explained in outcome PR01.

PR02.01 The focus here should be on equations using smaller numbers that can be more easily modelled or solved using concrete materials such as counters, pan balances, or pictures. This will enable students to build on their conceptual knowledge of one-step equations. Students will learn that there is more than one possible strategy that can be used to solve equations.

PR02.02, **PR02.03**, and **PR02.05** To provide reinforcement, demonstrate a simple two-pan balance scale with a numeric expression in each pan. For example, to solve $m + 5 = 14$, a student may solve the equation mathematically, outlining the steps that they did as they solved the equation using a pan balance.
$m + 5 = 14$
$m = 14 - 5$
$m = 9$

Remind students that since the scale is balanced, an equation can be written to represent the situation illustrated. Have students replicate the situation using blocks (centicubes) and a balance scale. Then, have them write the equation and the solution.

PR02.04 Students should be able to solve simple equations in the form $a + b = c$ where one of a, b, or c is missing (see below). Students should be able to explain the strategy they use.

$a + b = \square$	(e.g., $6 + 3 = \square$)	Join—Result Unknown
$a + \square = c$	(e.g., $2 + \square = 8$)	Join—Change Unknown
$\square + b = c$	(e.g., $\square + 4 = 5$)	Join—Initial Unknown
$c - a = \square$	(e.g., $7 - 2 = \square$)	Separate—Result Unknown
$c - \square = b$	(e.g., $4 - \square = 2$)	Separate—Change Unknown
$\square - a = b$	(e.g., $\square - 8 = 1$)	Separate—Initial Unknown

Similarly, students should be given the opportunity to solve a variety of simple equations for multiplication and division in the forms:

$a \times b = \square$	(e.g., $6 \times 3 = \square$)
$a \times \square = c$	(e.g., $2 \times \square = 8$)
$\square \times b = c$	(e.g., $\square \times 4 = 20$)
$c \div a = \square$	(e.g., $14 \div 2 = \square$)
$c \div \square = b$	(e.g., $12 \div \square = 2$)
$\square \div a = b$	(e.g., $\square \div 4 = 3$)

PR02.06 Students should be presented with a variety of problems involving each of the story structures for addition, subtraction, multiplication, and division. Encourage students to create problems for each of the four operations. It may be necessary to review the different types of equations such as the ones presented above.

It may be necessary to model for students how to create a problem for a given equation. As a whole class activity, use the equation, $46 + 12 = h$ to create a problem.

A possible problem could be, Bob has 46 hockey cards. Harry has 12 more hockey cards than Bob. How many cards does Harry have?

Another possible problem could be to provide students with a story problem, and ask them to record an equation that could be used to solve the problem.

- There are now 15 students in the classroom. 9 students went to choir. How many students are usually in the classroom?

  Students might represent this problem as $15 = n - 9$.

Measurement

SCO M01 Students will be expected to design and construct different rectangles, given a perimeter or an area or both (whole numbers), and make generalizations. [C, CN, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

M01.01 Draw two or more rectangles for a given perimeter in a problem-solving context.
M01.02 Draw two or more rectangles for a given area in a problem-solving context.
M01.03 Determine the shape that will result in the greatest area for any given perimeter.
M01.04 Determine the shape that will result in the least area for any given perimeter.
M01.05 Provide a real-life context for when it is important to consider the relationship between area and perimeter.

Performance Indicator Background

M01.01 and **M01.02** "Although measuring perimeter is often thought of as being different from linear measurement, it is actually only a variation. Measuring perimeter is an application of linear distance and students are measuring a linear distance that is not just a straight line." (Small 2013, 427) Students began in previous years using indirect measuring (i.e., using a string to measure around a shape, cutting the string, and then measuring the length of the string). Now students will measure directly and add the side lengths.

While investigating the distance around various rectangles, students should, in their own words, explain any generalizations noticed. They might conclude that the perimeter of a rectangle may be calculated in any of the following ways:

$l + w + l + w$
$2\ (l + w)$
$2l + 2w$

These all give perimeter measurement of a rectangle. Use of formulas for perimeter is not essential. The important thing is that students know that perimeter means distance around and have an accurate, efficient way to compute the perimeter of a given shape.

A common error with perimeter is that sometimes students forget to include the measures of unlabelled sides. Encourage students to label all sides before finding the perimeter. The following activity may be used to reinforce the above concepts.

- Have students work in pairs to solve the following problem: You need to determine the amount of fencing required to build a dog pen that has a length twice as long as its width. What are the dimensions of the pen? How can the perimeter of the pen be found without adding every length? You want to cover the floor of the pen with square paving stones. How many will you need? Find a way to calculate this without counting each stone.

Geo-boards or grid paper can be used to create various rectangles all with the same perimeter. For example, a rectangle with a perimeter of 20 units can have sides that are 8 cm, 8 cm, 2 cm, and 2 cm or 6 cm, 6 cm, 4 cm, and 4 cm or 7 cm, 7 cm, 3 cm, and 3 cm. Students should be working toward the realization that rectangles of different dimensions can have the same perimeter. Students should also determine the area of each of these rectangles to understand that though each of these rectangles has a perimeter of 20 units, the area of each of the rectangles is different.

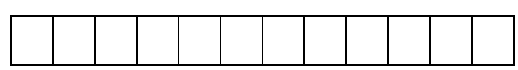
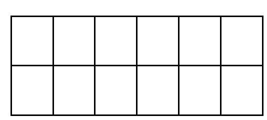
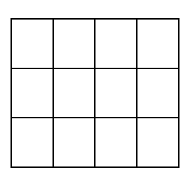
Through structured exploration activities, students should conclude that squares with the same area have the same perimeter and squares with the same perimeter have the same area. However, rectangles with the same area can have different perimeters, and rectangles with the same perimeter can have different areas. The generalization about squares is often over-generalized causing a common misconception about the relationship between area and perimeter in rectangles and in other polygons.

Students need to have many opportunities to experiment with developing their own strategy for calculating the perimeter and area of squares and rectangles. Elicit from students ways to find the areas of squares and rectangles.

M01.03 and **M01.04** This year the focus will be on working with area and perimeter when constructing rectangles. Students will be required to make conclusions regarding rectangular shapes that create the greatest and least areas. This investigation should be done using a problem-solving approach.

Students should be aware that if many rectangles can have the same area, then a longer length must accompany a shorter width. In fact, students may recognize that if one dimension is multiplied by any factor, the other dimension must be divided by that factor to retain the same area. This generalization may be made if you plan a guided investigation, asking students to draw specific rectangles to find the areas, to change the dimensions in specific ways, and to compare the new areas to the areas of the original rectangles, and the new dimensions to the original dimensions.

Students should also observe the relationship between area and perimeter of rectangles. They should notice that the perimeter of a rectangle increases as the difference between the length and width increases, and that conversely, the perimeter of a rectangle decreases as the difference between the length and width decreases. They should also observe that the greatest area and least perimeter is found when the rectangle is a square or close to a square. For example, students may be asked to construct rectangles with an area of 12 cm2 and to calculate the perimeter of each rectangle. They may create a chart such as the following:

Area	**Dimensions (length and width)**	**Perimeter**	**Picture**
12 cm^2	1 cm and 12 cm	26 cm	
12 cm^2	2 cm and 6 cm	16 cm	
12 cm^2	3 cm and 4 cm	14 cm	

M01.05 It is essential that the concepts of area and perimeter be applied to real-life situations. Realizing that to lay hardwood on a floor or to paint a wall takes knowledge of the area of the floor or wall, and that to determine requirements for fencing around a garden or yard takes knowledge of the perimeter of the space to be enclosed, lets students see the life applications of these mathematical concepts.

The playground can be a good place for students to investigate perimeter. First, ask students which unit of measurement they should use to measure the playground (mm, cm, or m). Then, have students estimate the perimeter by estimating the number of steps they would take if they walked around the perimeter. Record the estimates of each child. Using a trundle wheel, find the actual measurement of the perimeter.

Creating problems based on pieces of children's literature provides a spring board for thinking creatively about concepts like area and perimeter. After reading *Pigs* by Robert Munsch (1992), pose the following problem: A new pen is being built to corral the pigs after their adventures. The pen will be a rectangle, and 100 m of fencing must be used. Create three possible pens for the pig and model them on grid paper. Give the perimeter and area for each pen. Is it possible to create a pen that would have a shape that would be a square. Why might you choose this shape for the pen?

SCO M02 Students will be expected to demonstrate an understanding of measuring length (mm) by ▪ selecting and justifying referents for the unit millimetre (mm) ▪ modelling and describing the relationship between millimetre (mm) and centimetre (cm) units, and between millimetre (mm) and metre (m) units [C, CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

M02.01 Provide a referent for one millimetre, and explain the choice.
M02.02 Provide a referent for one centimetre, and explain the choice.
M02.03 Provide a referent for one metre, and explain the choice.
M02.04 Show that 10 millimetres is equivalent to one centimetre, using concrete materials.
M02.05 Show that 1000 millimetres is equivalent to one metre, using concrete materials.
M02.06 Provide examples of instances where millimetres are used as the unit of measure.
M02.07 Estimate and measure length in millimetres, centimetres, and metres.

Performance Indicator Background

M02.01, **M02.02**, **M02.03**, and **M02.06** Students should be able to explain that measures of distance are expressed using measurement units from the metre family. They should be able to give a referent for 1 cm and 1 m and should be developing referents for 1 mm. Students should be able to explain the relationship between the various units of measure and should use the relationships among the units to identify equivalent measures. For example, if students know that 1 metre is equivalent to 1000 mm, then they should be able to explain that 2000 mm is equivalent to 2 m. Students should be able to explain why some units are better to use in certain contexts than others.

Students should use personal referents, such as the width of their little fingers for a 1-centimetre benchmark. They could also measure their individual hand spans to establish another personal referent for centimetres. Since a ruler is a very common tool in the classroom, it can serve as a benchmark for 30 centimetres. The height of a door knob is often used as a benchmark for a metre. The metre stick provides an excellent referent for 1 metre and is a familiar object for students. Using these benchmarks, students should estimate lengths of objects from 1 cm to 100 cm or 1 m.

In discussing a millimetre, students realize that a millimetre is a very tiny linear measurement. Have students draw 1 mm on paper using a ruler. Ask students to brainstorm objects that are tiny and could be measured using millimetres. Examples may include the thickness of a fingernail or a button, width of an eyelash, width of the head of a straight pin, a fraction of a mosquito leg, etc.

M02.04 and **M02.05** The introduction to millimetres takes place after students have had experience using centimetres. A good way to introduce millimetres is to look at the length of objects that are between centimetres. For example, something that has a length of 25 mm has a length that falls between 2 cm and 3 cm. When discussing centimetres, use an overhead ruler that has only centimetres marked (if possible). Once students work with centimetres, introduce the ruler that includes the markings for centimetres and millimetres.

Students should measure distances using millimetres, centimetres, and metres in contexts where they arise. This is the first exposure to millimetres, so careful development of these concepts needs to take place. Students should see that millimetres can be found by dividing a centimetre into ten equivalent parts. They should be exposed to measuring tools that show the mm and cm relationship.

Rulers and metre sticks are good models for exploring the relationship between the units millimetres and centimetres, centimetres and metres, and millimetres and metres. A metre stick is divided into centimetres, so students can see that 100 cm is equal to 1 metre. A metre stick is also divided into millimetres, so students can see that 1000 mm is equal to 1 metre. A ruler can be used to show that 1 cm is equal to 10 mm. If 1 cm equals 10 mm, and if 100 cm is equal to 1 metre, then 1000 mm must equal 1 metre. Thus, 1000 mm = 100 cm = 1 m.

Students should notice the relationship between the units as they move from millimetres to centimetres, millimetres to metres, and centimetres to metres. They should also notice there is a similar pattern to tens as in the place-value chart. For example, 10 mm are needed to make 1 cm, 1000 mm are needed to make 1 m, and 100 cm are needed to make 1 m. It should also be noted that 1 mm is one-tenth of 1 cm and one-thousandth of 1 m and that 1 cm is one-hundredth of a metre.

It is important for students to understand that the unit chosen for measurement affects the numerical value of the measurement. The larger the unit the smaller the numerical value. For example, 1 m = 100 cm, the larger unit, which is metres, has the numerical value of 1, but the distance measured in a smaller unit such as centimetres yields the larger numerical value of 100.

Students are developing understanding of the benchmarks for millimetre, centimetre, and metre. They are now ready to use these benchmarks to develop estimation competency. If students have established benchmarks against which they are visually comparing other lengths, they are less likely to give "wild guesses" or to "pull numbers out of the air." In the initial reinforcements, after students arrive at their estimates, they should get the actual measure using a ruler or metre stick, and discuss the strategies used to get the estimates and the reasonableness of their estimates. This will help them refine their estimating abilities. However, with continued practise, they should not determine the actual measure. Students should understand that they will often estimate in real life when an estimate is all that is needed. At other times, they will estimate to be alert to the reasonableness of measures they find or are given.

In order to introduce estimation, assemble a number of objects and prepare a list of ranges for estimates of the lengths (8–10 questions at a time would be effective). For example, display a wastebasket and ask, Will the height of a wastebasket be about 35–45 cm or 20–30 cm? Will the length of a piece of lined paper be about 10–20 cm or 20–30 cm? After each question, invite students to share how they decided the most reasonable range. This work should be extended to estimating perimeters.

SCO M03 Students will be expected to demonstrate an understanding of volume by
- selecting and justifying referents for cubic centimetre (cm^3) or cubic metre (m^3) units
- estimating volume using referents for cubic centimetre (cm^3) or cubic metre (m^3)
- measuring and recording volume (cm^3 or m^3)
- constructing rectangular prisms for a given volume

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

M03.01 Identify and explain why the cube is the most efficient unit for measuring volume.
M03.02 Provide a referent for a cubic centimetre, and explain the choice.
M03.03 Provide a referent for a cubic metre, and explain the choice.
M03.04 Determine which standard cubic unit is represented by a given referent.
M03.05 Estimate the volume of a given 3-D object using personal referents.
M03.06 Determine the volume of a given 3-D object using manipulatives, and explain the strategy.
M03.07 Construct a rectangular prism for a given volume.
M03.08 Construct more than one rectangular prism for a given volume.

Performance Indicator Background

M03.01 In order for students to identify the cube as the most efficient unit for measuring volume, they have to be exposed to measuring volume using objects like marbles or styrofoam peanuts. A suggestion would be to have groups of two to three students determine the volume of a given box by first filling it with marbles and recording this number. Then have them fill the box with cubes, record the number, and compare and explain the differences. Involving students in a hands-on activity using cubes to fill boxes will help students understand why the volume is recorded in standard cubic units.

M03.02, **M03.03**, and **M03.04** Students should develop personal referents for cubic centimetres and cubic metres. The use of personal referents helps students establish the relationships between the units. Students should realize that a cube with a measure of 1 centimetre on each edge has a volume of 1 cubic centimetre, and a cube with a measure of 1 metre on each edge has a volume of 1 cubic metre. A good way to model a cubic metre as a personal referent is to take 12 newspaper rolls (1 metre long each) and tape together to form a cube. If the cube were solid, it would have a volume of 1 m^3. Students will be able to see that the newspaper cube has a height, a length, and a width of one metre each.

M03.05 Students should estimate volume using a variety of different sized boxes. For estimation of volume in Mathematics 5, student competency would be the ability to recognize a reasonable estimate of the volume of objects in cubic centimetres or cubic metres. When given an estimate, students should judge its reasonableness by visualizing the number of small base-ten cubes or cube constructed from newspaper (see above) this estimate represents, and comparing the object to this visualized volume. Students would thus be using the volume of a small cube from the base-ten unit blocks as a referent for one cubic centimetre (1 cm^3), and the large newspaper cube as a referent for one cubic metre (1 m^3).

M03.06, **M03.07**, and **M03.08** Students should learn to quantify volume by determining the total number of units the 3-D region contains. The volume units that students will generally encounter are cubic centimetres (cm^3) and cubic metres (m^3). If students explore the size of a million (see outcome N01) by building a cubic metre with metre sticks, they will have a good mental image of one cubic metre

(m^3). Students should be able to explain that a cubic centimetre (cm^3) is a measure of volume using a cube that is 1 cm on each side. Students should connect this with a metric linking cube or the small cube from the base-ten materials. Students should also make use of the large cube from the base-ten materials as the large cube is 10 cm on each side and has a volume of 1000 cm^3. Students should have a sense of which unit is the most appropriate volume unit to use in any circumstance. Ask students to measure the volume of large and small rectangular prisms by counting the number of cubes it takes to build a duplicate of each prism.

M03.07 and **M03.08** After students have the understanding that a one-centimetre cube has a volume of 1 cm, they should build various rectangular prisms with centimetre cubes and then determine the volume of the prism. Through these construction activities, students should understand that more than one rectangular prism is possible for a given volume.

SCO M04 Students will be expected to demonstrate an understanding of capacity by ▪ describing the relationship between millilitre (mL) and litre (L) units ▪ selecting and justifying referents for millilitre (mL) and litre (L) units ▪ estimating capacity using referents for millilitre (mL) and litre (L) ▪ measuring and recording capacity (mL or L) [C, CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

M04.01 Demonstrate that 1000 millilitres is equivalent to one litre by filling a one-litre container using a combination of smaller containers.
M04.02 Provide a referent for one litre, and explain the choice.
M04.03 Provide a referent for one millilitre, and explain the choice.
M04.04 Determine the capacity unit of a given referent.
M04.05 Estimate the capacity of a given container using personal referents.
M04.06 Determine the capacity of a given container using materials that take the shape of the inside of the container (e.g., a liquid, rice, sand, beads), and explain the strategy.

Performance Indicator Background

M04.01 Capacity units that students will explore include millilitres (mL) and litres (L). The particular relationships that students should understand include 1 L is 1000 mL and 1 mL is 0.001 L. Provide students with a variety of 1-L containers and containers with varying capacities of less than one litre. Ask students to use the smaller containers to fill the 1-L container, charting the amounts added until the 1-L container is filled. When students determine the sum of the amounts they added to each of the 1-L containers, they should realize that it takes 1000 mL to equal 1 L. This activity can also be done in reverse, starting with a filled litre container. Rote conversion of millilitres to litres or litres to millilitres is not expected.

M04.02 and **M04.03** Students should develop personal referents for these units. A useful referent for a millilitre would be a unit base-ten cube. Use a medicine dropper that shows a 1-mL marking to help students visualize 1 mL. Since the millilitre is so small, students should use also have referents for other capacities involving millilitres. Graduated measuring cups can be used to provide students with a sense of 25 mL, 100 mL, 250 mL, and 500 mL. Graduated medicine cups that typically accompany students' medication or medicine droppers provide good examples of things measured using a small number of millilitres such as 15 mL. Common measurements used in cooking include 5 mL, which is about the same as a teaspoon or 15 mL, which is about the same as a tablespoon. Talk about how small babies often receive medicine in these units. Also, eye and ear drops are often given in quantities even less than a millilitre. Personal referents for 1 L may include a 1-L milk or juice container, 1-L ice cream container, or a 1-L water bottle.

The use of personal referents will help students establish the relationships between the units—the small cube in the base-ten materials has a volume of 1 cm^3 and holds 1 mL and the large cube has a volume of 1000 cm^3 and holds 1 L. These referents help students estimate and recognize the reasonableness of any conclusion.

M03.04 Using a series of different containers (e.g., a glass of milk, a bathtub, the punch in a punch bowl, or a container of laundry detergent) ask students if the capacity of each container would be better measured in millilitres or litres. Ask them to justify their reasoning. Then, they would estimate the capacity in the chosen unit and then measure the capacity to check their estimate.

M04.05 For capacity estimation, student competency should centre on the reasonableness of given estimates and on making reasonable decisions involving capacities (how much a container is capable of holding) of a variety of commonly used containers, such as pop cans and bottles and juice and milk containers. Student could use a small cube from the base-ten unit blocks as a benchmark for 1 mL, a rod as a benchmark for 10 mL, a flat as a benchmark for 100 mL, and a large cube as a benchmark for 1 L.

Of all the metric units, millilitres and litres are the ones most commonly used in general society; therefore, students will likely bring out-of-school experiences to these estimation activities.

Students should have a sense of which unit is the most appropriate capacity unit to use in any circumstance. For example, students should chose the unit of measurement that would be used for a variety of situations, such as filling a sink, pouring a glass of milk, buying gasoline, determining the amount of air in a room, and getting a flu shot.

M04.06 Invite groups of students to investigate the capacities of various beverage containers to determine which size container is found most often. They might record their findings in a graph or table and present it to the class.

M04.07 Students should have many experiences determining the capacity of given containers using materials that take the shape of the inside of the container (e.g., a liquid, rice, sand, beads). Students should use graduated measuring cups, graduated cylinders, or other smaller containers whose capacity is known to measure the materials to determine the capacity of the given containers.

Geometry

SCO G01 Students will be expected to describe and provide examples of edges and faces of 3-D objects, and sides of 2-D shapes that are parallel, intersecting, perpendicular, vertical, and horizontal. [C, CN, R, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

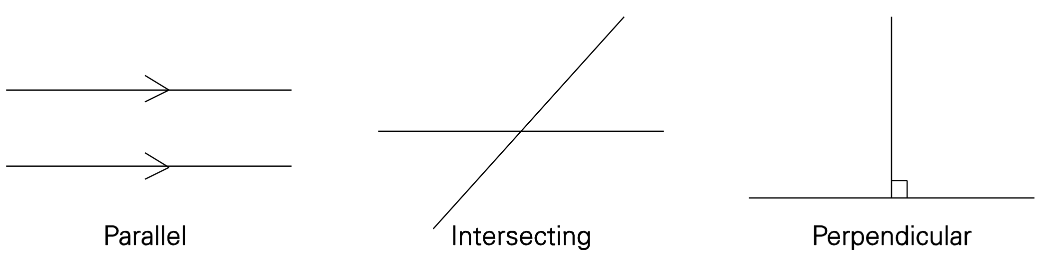
Performance Indicators

G01.01 Identify parallel, intersecting, perpendicular, vertical, and horizontal edges and faces on 3-D objects.
G01.02 Identify parallel, intersecting, perpendicular, vertical, and horizontal sides on 2-D shapes.
G01.03 Provide examples from the environment that show parallel, intersecting, perpendicular, vertical, and horizontal line segments.
G01.04 Find examples of edges, faces, and sides that are parallel, intersecting, perpendicular, vertical, and horizontal in print and electronic media, such as newspapers, magazines, and the Internet.
G01.05 Draw 2-D shapes that have sides that are parallel, intersecting, perpendicular, vertical, or **horizontal.**
G01.06 Build 3-D objects that have edges and faces that are parallel, intersecting, perpendicular, vertical, or horizontal.
G01.07 Describe the faces and edges of a given 3-D object using terms such as **parallel**, **intersecting**, **perpendicular**, **vertical**, or **horizontal**.
G01.08 Describe the sides of a given 2-D shape using terms such as **parallel**, **intersecting**, **perpendicular**, **vertical**, or **horizontal**.

Performance Indicator Background

G01.01, **G01.02**, **G01.03**, **G01.04**, **G01.07**, and **G01.08** These indicators overlap, and one activity may cover several indicators. This outcome can be introduced by discussing the definitions and identifying examples in the classroom. Students should explore the sides of 2-D shapes, and the edges and faces on 3-D objects. They should have opportunities to describe and provide examples of 2-D shapes whose sides are parallel, intersecting, perpendicular, vertical, and horizontal, and 3-D objects whose faces and edges are parallel, intersecting, perpendicular, vertical, and horizontal.

Lines in the same plane can be parallel or they can intersect. Parallel lines never meet since they remain a constant distance apart. Whenever two lines intersect, they meet at a single point. Perpendicular lines are intersecting lines that form a right angle (a square corner or 90 degrees).



To develop the concepts of vertical and horizontal, have students identify examples inside and outside the classroom. To get started, they could consider the horizon. Which way is the horizon? Up and down or left to right?

Students have been introduced to the concept of edges and faces in earlier grades. Faces are the flat surfaces of a 3-D object. Edges are where two faces intersect. Adjacent faces of a cube are perpendicular and opposite faces are parallel.

G01.04 To provide examples from the environment that show parallel, intersecting, perpendicular, vertical, and horizontal line segments, faces, or edges, consider going on a walk to explore the different shapes and objects around your community. It is important that students record their observations. Students may also be provided with magazines, newspapers, photographs, and pre-selected internet sites in order to find parallel, intersecting, perpendicular, and vertical and horizontal lines. Using a chart with each of the concepts as a heading may simplify the activity.

G01.05 Students may need to be reminded to always use a ruler when drawing straight lines.
To draw a 2-D shape with parallel lines, students can use their rulers to measure equal distances between lines.

For perpendicular lines, remind students they are drawing a right angle (i.e., a 90-degree angle). A simple index card can be used to draw perpendicular lines (right angles) and to draw straight lines.

Students may use a Mira to draw perpendicular lines. By drawing a line segment and placing the Mira across it, and moving the Mira until the image of the part of the segment on one side of the Mira falls on the segment on the other side, the line on which the Mira sits is perpendicular to the original line. Therefore, the perpendicular line can be drawn by tracing the beveled edge of the Mira with a pencil. When the image of the endpoint of the segment falls on the other endpoint, the Mira is bisecting the segment; therefore, the line on which the Mira sits will be the perpendicular-bisector of the segment. In addition to work with the Mira, students can use a variety of manipulatives to investigate parallel and perpendicular lines, including straws, toothpicks, and geo-strips.

Working with the quadrilateral family (outcome G02) will provide students with a range of experiences to identify, compare, and analyze the properties of polygons. Students should use Miras to investigate and compare the perpendicular lines, bisectors of line segments, and perpendicular-bisectors related to 2-D shapes.

G01.06 Activities related to 3-D objects may focus on, but are not limited to, rectangular and triangular prisms, which were explored in Mathematics 4. Teachers may include pyramids to let students see that not all solids have parallel faces. This activity can include other prisms, for example, hexagonal or octagonal. Prisms, by definition, have two congruent parallel faces made of polygons called bases with line segments joining corresponding points on the two bases. These line segments are always parallel and are called edges.

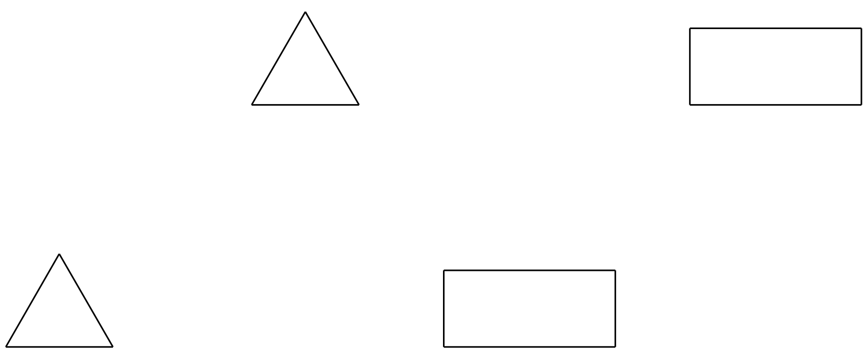
In order to investigate the faces and edges of 3-D objects, hold up a cereal box (rectangular prism). Ask students to identify the faces and edges. Lead a discussion that will have students describe edges and faces in terms of parallel, intersecting, perpendicular, vertical, and horizontal. Then, have students work in pairs. One student chooses a geometric solid and describes it according to its attributes. The second student then tries to identify the solid. Once the solid is identified, students switch roles.

Students should use correct mathematical language to describe 2-D shapes and 3-D objects. Another way to have students explore the edges and faces of 3-D objects is to have them work in small groups to stack pattern blocks to build prisms. Stacked pattern blocks will form triangular prisms, rectangular prisms, trapezoidal prisms, rhomboidal prisms, and hexagonal prisms. After students have constructed the prisms, discuss the following questions:

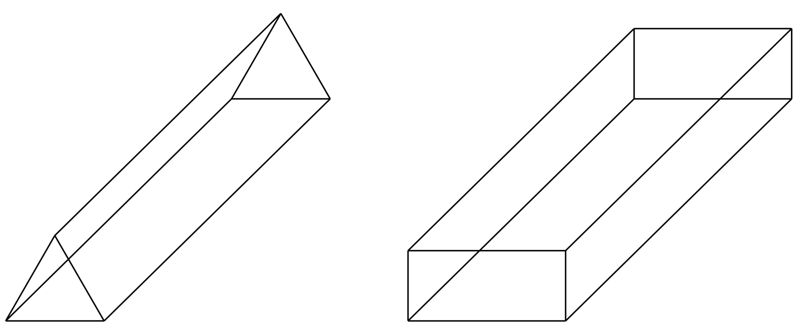
- Which solid has the most parallel faces?
- Which solid has the least number of edges?
- Which solid has only two parallel faces?
- Which solids have eight intersecting edges?
- Which solid has four sets of parallel faces?

Drawing 3-D objects will be new to students and may need to be addressed as a separate mini-lesson.

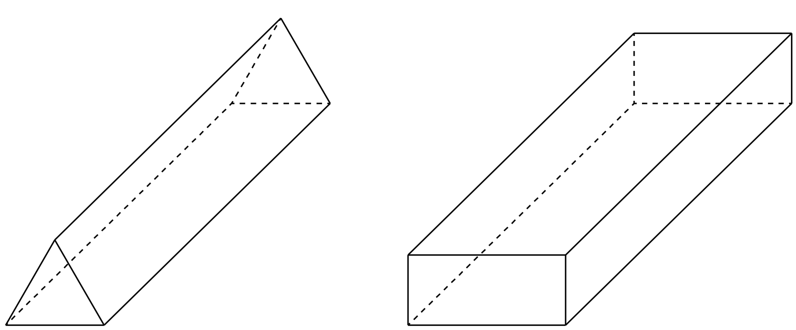
Step 1: Draw two congruent polygons (triangles or rectangles) slightly staggered vertically.



Step 2: Join corresponding vertices with parallel lines.



Step 3: Erase “unseen” lines and replace with dotted lines.



SCO G02 Students will be expected to name, identify, and sort quadrilaterals, including rectangles, squares, trapezoids, parallelograms, and rhombi, according to their attributes. [C, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

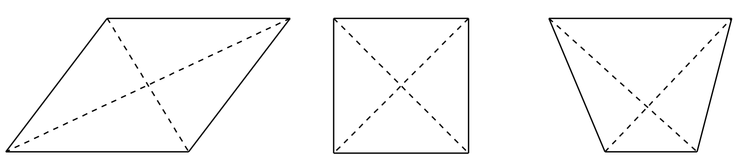
G02.01 Identify and describe the characteristics of a pre-sorted set of quadrilaterals.
G02.02 Sort a given set of quadrilaterals, and explain the sorting rule.
G02.03 Sort a given set of quadrilaterals according to the lengths of the sides.
G02.04 Sort a given set of quadrilaterals according to whether or not opposite sides are parallel.
G02.05 Sort a set of quadrilaterals based on properties such as diagonals are congruent, diagonals bisect each other, and opposite angles are equal.
G02.06 Name and classify quadrilaterals according to their attributes.

Performance Indicator Background

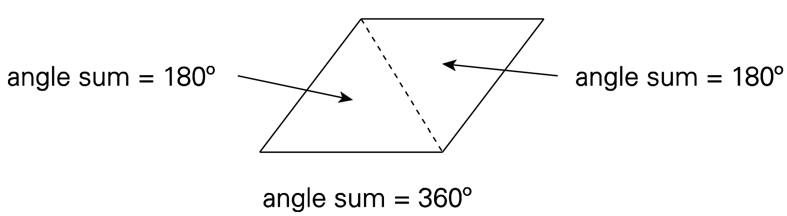
G02.01 Initially, students identify shapes by their overall appearance. While many of their properties have been implied, it is now the goal to make the properties of some shapes explicit. There will be a particular focus in this grade on developing an understanding of the nested structure of the quadrilateral family. Squares, rectangles, parallelograms, rhombi, and trapezoids should each be analyzed separately so that students, through hands-on investigations, can see and describe the properties of these quadrilaterals.

Quadrilaterals are four-sided polygons. Although rectangles are the most common quadrilaterals that you see in everyday life, students will soon discover that there are many classes of quadrilaterals. The quadrilateral family includes squares, rectangles, rhombi, parallelograms, and trapezoids, along with other four-sided non-regular shapes.

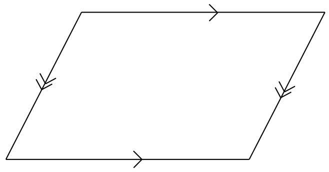
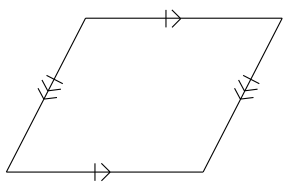
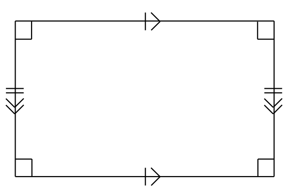
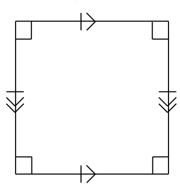
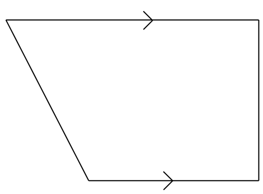
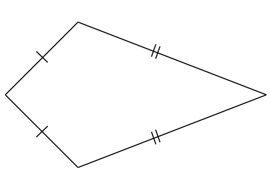
All quadrilaterals are polygons with four straight sides and four vertices (four angles). Every quadrilateral has two diagonals.



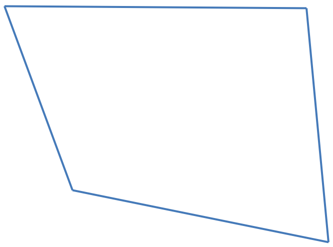
Because a diagonal divides a quadrilateral into two triangles, the sum of the angles in a quadrilateral is always 180 + 180 = 360.



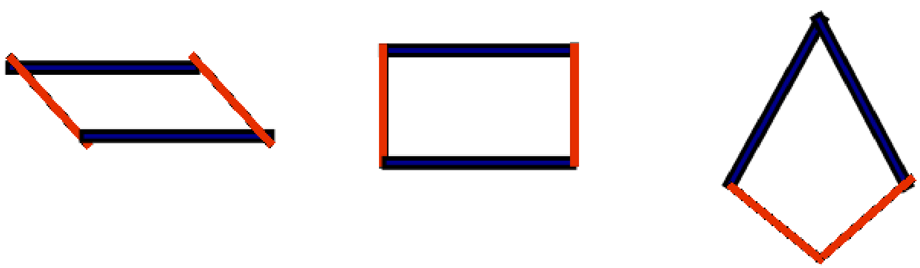
The properties of special types of quadrilaterals are listed in the chart below:

Quadrilateral	**Properties**
Parellelogram 	A quadrilateral with two pairs of parallel sides.
Rhombus 	A parallelogram with all sides equal in length.
Rectangle 	A parallelogram with four right angles.
Square 	A rectangle with all sides equal in length.
Trapezoid 	A quadrilateral with one pair or parallel sides. If two sides are equal, it is an isosceles trapezoid.
Kite 	A quadrilateral with two pairs of equal adjacent sides.

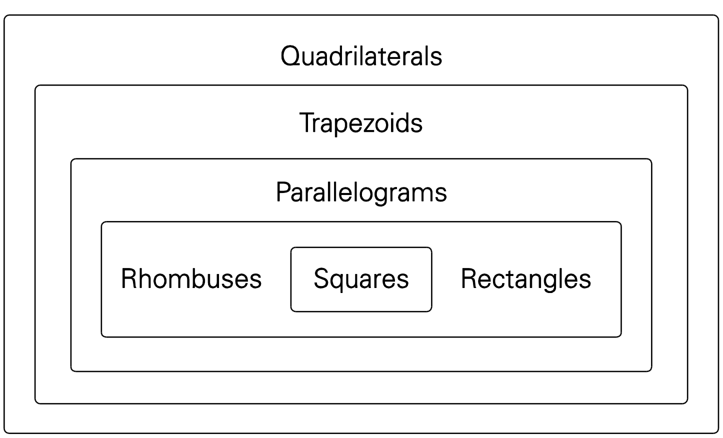
Some quadrilaterals do not fit into any of the classifications listed in the chart.



After students have identified the properties of quadrilaterals, they should be engaged with “if then” reasoning statements to justify classification of quadrilaterals. For example, a student might say, “If it has four square corners, it is a rectangle,” or “If I know opposite sides are parallel, then it is a parallelogram.” Students should use concrete materials such as geo-strips to build shapes based on given criteria such as, I have four equal sides, what might I be? or I have two pairs of equal sides, who might I be? and so on. Students should try to create as many shapes as possible to match a given criteria so that they see the variety of examples that can fit given statements. For example, each of the shapes below have two pairs of equal sides.



Students should have opportunities to explore and investigate properties of quadrilaterals. Using concrete materials such as geo-strips, pipe cleaners, etc., students can build a variety of four-sided figures and describe the properties of the various shapes. Students should make generalizations about the length and parallelism of sides, and the size of the contained angles (right, greater than right, less than right, congruent). Students should explain classification within the quadrilateral family based on properties of shapes. For example, they could explain why a rectangle could also be called a parallelogram, or why a square can be part of the rectangle family or the rhombus family. Students should develop a graphic representation to show the nestedness of the quadrilateral families.



G02.02, **G02.03**, **G02.04**, and **G02.06** The purpose of sorting shapes is to focus attention on the particular geometric attributes shared by a type of shape. For example, students might use Caroll Diagrams to group polygons with particular attributes like number of sides, types of angles, or measures of either sides or angles. Likewise, they might use a Venn diagram to sort shapes by two or more attributes. In both cases, the opportunity to work with physical models allows students to develop an understanding of shape based on their own observations, rather than a list of prescribed definitions or formulas.

G02.05 Students should identify and sort quadrilaterals, including rectangles, squares, trapezoids, parallelograms, and rhombi, according to their attributes. They should use their knowledge of right angles (outcome G05) to describe the angles in a shape.

Guided investigations using paper folding, Miras, and direct/indirect measurements of lengths and angles will enable students to notice and discuss the patterns regarding diagonals in quadrilaterals; these patterns will be the diagonal properties of these shapes. From investigations like these, students should conclude that the diagonals of a

- square are equal in length, bisect each other, and intersect to form four right angles (This also means that the diagonals are perpendicular-bisectors of each other. The diagonals form four congruent triangles.)
- rectangle are equal in length, bisect each other, and form two pairs of congruent triangles
- rhombus are perpendicular bisectors of each other, form four congruent triangles, and are the two lines of reflective symmetry of the rhombus
- parallelogram bisect each other and form two pairs of congruent triangles (They should also conclude that there are no special properties of its diagonals.)

These properties should be developed for each figure, applied in a variety of ways, compared to the others, and combined with the side and angle properties of the shapes.

SCO G03 Students will be expected to perform a single transformation (translation, rotation, or reflection) of a 2-D shape (with and without technology) and draw and describe the image. [C, CN, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

G03.01 Translate a given 2-D shape horizontally, vertically, or diagonally, draw the image, and describe the position and orientation of the image.
G03.02 Rotate a given 2-D shape about a vertex, draw the image, and describe the position and orientation of the image.
G03.03 Reflect a given 2-D shape in a line of reflection, draw the image, and describe the position and orientation of the image.
G03.04 Perform a transformation of a given 2-D shape by following instructions.
G03.05 Draw a 2-D shape, translate the shape, and record the translation by describing the direction and magnitude of the movement.
G03.06 Draw a 2-D shape, rotate the shape about a vertex, and describe the direction of the turn (clockwise or counter-clockwise) and the fraction of the turn (limited to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or full turn).
G03.07 Draw a 2-D shape, reflect the shape, and identify the line of reflection and the distance of the image from the line of reflection.
G03.08 Predict the result of a single transformation of a 2-D shape and verify the prediction.

Performance Indicator Background

No additional background.

SCO G04 Students will be expected to identify and describe a single transformation, including a translation, rotation, and reflection of 2-D shapes. [C, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

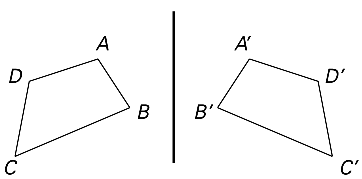
Performance Indicators

G04.01 Provide an example of a translation, rotation, and reflection.
G04.02 Identify a given single transformation as a translation, rotation, or reflection.
G04.03 Describe a given rotation about a point of rotation by the direction of the turn (clockwise or counter-clockwise).
G04.04 Describe a given reflection by identifying the line of reflection and the distance of the image from the line of reflection.
G04.05 Describe a given translation by identifying the direction and magnitude of the movement.
G04.06 Identify transformations found in everyday pictures, art, or the environment.

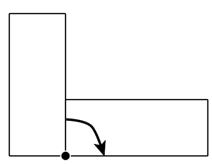
Performance Indicator Background

G04.01 Students need to be exposed to numerous examples of each of the transformations to recognize when one has been performed. See also performance indicator background for outcome G03.

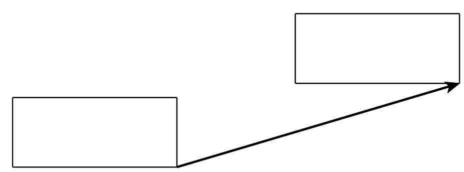
Example of a reflection (flip)



Example of a rotation (turn)



Example of a translation (slide)



G04.02 and **G04.03** Students have no prior experiences with rotations. As well, students will only be expected to rotate the shape about a vertex at this grade level.

A rotation moves shapes in a circular motion. Rotations are commonly the most challenging of the transformations. Students need many first-hand experiences making rotations and examining the results before they will be able to identify such rotations given to them. At this grade level, the emphasis should be on drawing rotation images and identifying a rotation image with centres on one of the vertices and angles that are $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ turns.

When students first begin working with turns, they identify and describe them in terms of fractions of a circle $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ turn. In adding to describing the amount of turn, students also need to identify the turn direction (clockwise or counter-clockwise). Clockwise and counter-clockwise are abbreviated as "cw" and "ccw."

G04.02 and **G04.04** Students need to make the connection between their prior knowledge of symmetry and the line of reflection. The line of reflection creates symmetry between the pre-image and image, whereas a line of symmetry typically refers to symmetry within a given object.

A reflection can be identified if

- a 2-D shape and its image are congruent
- a 2-D shape and its image are of opposite orientation (This is, if we go around the object *ABCD* in a clockwise direction, the image *A'B'C'D'* would require a counter-clockwise direction.)

Model, using mathematical language, how to describe a given reflection. For example, the image has been reflected in a horizontal or vertical line of reflection.

G04.02 and **G04.05** The general properties students should use to identify translations are

- the 2-D shape and its image are congruent
- the 2-D shape and its image have the same orientation (That is, if we go around the object *ABCD* in a clockwise direction, we should be able to also go around its image *A'B'C'D'* in a clockwise direction.)

Model, using mathematical language such as two units right and three units down, how to describe given translations.

SCO G05 Students will be expected to identify right angles. [ME, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

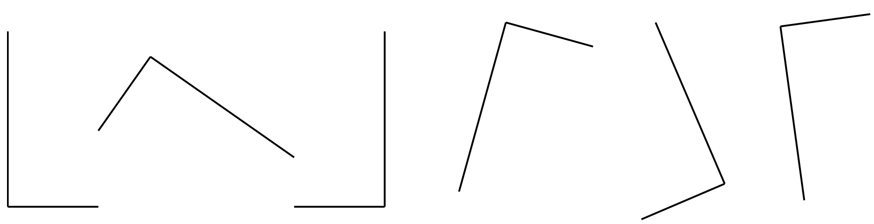
Performance Indicators

G05.01 Provide examples of right angles in the environment.
G05.02 Sketch right angles without the use of a protractor.
G05.03 Label a right angle, using a symbol.
G05.04 Identify angles greater than or less than a right angle.

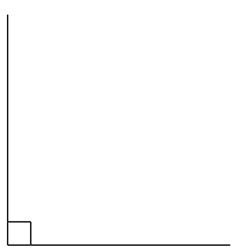
Performance Indicator Background

G05.01 Students should be introduced to the concept of right angles as a descriptor for what they may have referred to as a square corner. Classroom objects provide many examples of right angles, and these should be explored, drawn, and discussed.

G05.02 Students may use the corner of an index card to aid them in sketching right angles. Students should sketch angles in a variety of positions on the page, as shown below.



G05.03 Students should identify and label right angles in shapes using a symbol.



G05.04 Students are not measuring angles in degrees at this stage. They are comparing angles by sight. After they have been introduced to the concept of right angles, they should be able to compare a given angle to a right angle and should describe the given angle as more than a right angle or less than a right angle. Students should examine the angles of a variety of polygons to determine if the angles are right, less than right, or more than right angles.

Students could be provided with index cards and could identify the right angles on the corners of these cards. Then, students could use the card as a right angle checker. Students could be asked to use the right angle checker to find right angles in various 2-D shapes. For angles that are not right angles, students could describe them as less than a right angle or more than a right angle.

Statistics and Probability

SCO SP01 Students will be expected to differentiate between first-hand and second-hand data. [C, R, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

SP01.01 Explain the difference between first-hand and second-hand data.
SP01.02 Formulate a question that can best be answered using first-hand data and explain why.
SP01.03 Formulate a question that can best be answered using second-hand data and explain why.
SP01.04 Find examples of second-hand data in print and electronic media, such as newspapers, magazines, and the Internet.

Performance Indicator Background

SP01.01 and **SP01.04** Students need to be given opportunities to activate their background knowledge of keeping a tally and creating bar graphs. Discuss and record various kinds of situations that warrant data collecting.

Introduce the key terms of **first-hand data** and **second-hand data**. First-hand data is collected by the researcher (in this case the students) and is best used when they are looking for answers to questions about people, places, or objects found in their everyday lives. First-hand data is required when this information is not readily available from existing respectable sources. It is also used when data is limited or when students are just beginning to learn about data. It will be necessary to review first-hand data techniques such as surveys, observations, interviews, and experiments.

Second-hand data is data that has been collected by someone else. Second-hand data can be found in print and on the Internet. Some secondary sources include the following:

- *The World Almanac for Kids*
- Statistics Canada
- Guinness World Records
- *World Almanac and Book of Facts*

Second-hand data sources can also consist of newspapers and resource books. Further explore curriculum areas such as science and social studies in investigating second-hand data.

Provide students with examples of first-hand and second-hand data and ask them to identify the type of data (i.e., a bar graph showing hockey statistics found in the newspaper represents a source of data for secondary analysis, while asking students to survey the class about their preference in popcorn seasonings is an example of first-hand data). Encourage students to reflect on the meaning of first-hand and second-hand data and record this reflection in their journals.

Have a class discussion on the two types of data. Guide your discussion according to the following information:

First-hand Data

- Data collected by the researcher (in school, this is the student)
- Collection methods include observations, surveys, and experiments
- Primary source of data for students
- Questions created should help give more precise answers

Second-hand Data

- Data collected by others and used for secondary analysis
- Found in the news, on the Internet, in statistics
- Students are not part of data collections or questioning
- Create questions based on data

SP01.02 and **SP01.03** "The process of data analysis begins with the formulation of questions concerning an issue or topic of interest. Students should be encouraged to formulate questions that address issues in their everyday lives at school, home, or within their communities. All data investigations begin with questions, yet asking good questions is a skill that takes time to develop." (Chapin et al. 2002, 11)

Brainstorm questions that can be best answered by using first-hand data. Some examples are as follows:

- What kind of ice cream do you prefer?
- What is your favourite type of music?

Compile a list of student-generated questions to display on a bulletin board. As students take part in the teacher-student dialogue, they should discuss how their question constitutes a good question. These questions will then be posted on the bulletin board. Discuss with students the importance of posing specific questions that provide a clear answer (i.e., What is your favourite music? is not as specific as What is your favourite type of music?) Consider questions like, How many cars are in the parking lot on an average day (observational)? Or, What number occurs most often when you roll two dice and add the numbers together (experimental)?

The Internet provides a wealth of data about sports, world records, and Canadian statistics that can be used for secondary analysis. Based on the data from other sources, students should pose questions that allow them to do secondary analysis of their data. Provide students with examples of data from a variety of sources (print or electronic) . Ask students to create questions based on the data and then share with their classmates.

SCO SP02 Students will be expected to construct and interpret double bar graphs to draw conclusions. [C, PS, R, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

SP02.01 Determine the attributes (title, axes, intervals, and legend) of double bar graphs by comparing a given set of double bar graphs.
SP02.02 Represent a given set of data by creating a double bar graph, label the title and axes, and create a legend without the use of technology.
SP02.03 Draw conclusions from a given double bar graph to answer questions.
SP02.04 Identify examples of double bar graphs used in a variety of print and electronic media, such as newspapers, magazines, and the Internet.
SP02.05 Solve a given problem by constructing and interpreting a double bar graph.

Performance Indicator Background

SP02.01 and **SP02.02** Prior to Mathematics 5, students created and labelled bar graphs using appropriate scales and appropriate attributes.

A double-bar graph shows how two different sets of data are alike or different. Using a legend helps the reader interpret a double-bar graph. Through the use of sport statistics, a connection can be made between some students' out-of-school interests and the area of mathematics. Hockey, soccer, baseball, and football statistics lend themselves to the construction of a double-bar graph.

Remind students that in a double-bar graph, each set of data must use the same scale, have a title, scale, and legend and the order of data must remain the same throughout. Provide students with examples of various graphs displaying and/or describing the above attributes.

Invite students to discuss the following questions to draw a conclusion:

- What have I learned from this graph?
- What conclusions can you gather from this data?
- What message is conveyed in this double-bar graph?
- Who did the collecting?
- Who was the data collected for?
- What message is the data telling us?

Model the construction of a double-bar graph before students work independently to construct their own. Teachers may use chart paper grid pads in constructing the double-bar graphs. At the beginning, students could use grid paper to construct bar graphs to ensure that the squares are all of equal size.

SP02.03 It is important to note to students that graphs can be factual; that is, the information can be read directly from the graph or can provide an opportunity to make inferences that are not directly seen or observed.

SP02.04 Ask students to locate examples of double-bar graphs found in newspapers, magazines, pamphlets, the Internet, posters, or books. Discuss the different attributes found on these graphs.

SP02.05 Students should construct a double-bar graph to assist them in solving a given problem. For example, students could collect data and construct a double-bar graph to answer questions such as the following:

- Which grade-level students seem to enjoy mathematics the most?
- Which students watch NHL hockey the most, boys or girls?
- Students could then draw conclusions based on their graphs.

SCO SP03 Students will be expected to describe the likelihood of a single outcome occurring, using words such as **impossible**, **possible**, and **certain**. [C, CN, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

SP03.01 Identify examples of events from personal contexts that are impossible, possible, or certain.
SP03.02 Classify the likelihood of a single outcome occurring in a probability experiment as impossible, possible, or certain.
SP03.03 Design and conduct a probability experiment in which the likelihood of a single outcome occurring is impossible, possible, or certain.
SP03.04 Conduct a given probability experiment a number of times, record the outcomes, and explain the results.

Performance Indicator Background

SP03.01 Position the three reference points **impossible**, **possible** and **certain** on a clothesline. Provide examples of events that would be impossible, possible, or certain, such as

- I will walk to the moon next week in my pyjamas.
- My eight-month old baby sister will drive me to school.
- I will go for a walk after supper.
- I will ride my bike for 30 minutes today.
- I will go to school on Sunday.

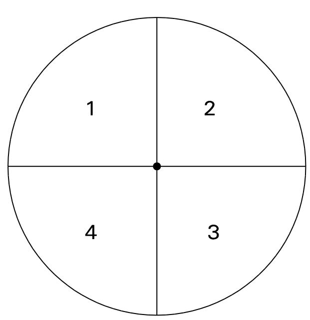
Have students place these events on the clothesline (probability line) in the appropriate places and explain their thinking. Students could also create their own probability events and invite other classmates to place them on the probability line.

SP03.02 Experimental probability is the probability reached by actually performing an experiment. For example, in theory, if you flip a coin two times, you should get one head and one tail. However after flipping the coin twice you may get two heads. The more you flip the coin, the greater the chance of having equal heads and equal tails.

Using spinners, dice, or coloured cubes, have students predict whether the outcome will be impossible, possible, or certain. Examples include the following:

- In a bag with eight red cubes and four yellow cubes, a red cube is more possible to be drawn than a yellow cube.

- Using the spinner below, it is impossible to spin a 5 and possible to spin a 1, 2, 3, or 4.



- When rolling a die, it is certain that they will roll a 1, 2, 3, 4, 5, or 6, but impossible to roll a 7.

SP03.03 Students design and then conduct probability experiments in which the likelihood of a single outcome occurring is impossible, possible, or certain. Two suggestions for activities are as follows:

- Invite students to place coloured cubes in a bag to create a situation/outcome where choosing a red cube is
  - certain (all cubes red)
  - possible (at least one cube is red)
  - impossible (no red cubes)
- Have students invent a game that is related to sums and products using
  - dice, spinners, or cards (i.e., You get a point if the sum of the cards you pick is highest. Then students will try to decide if the games are fair).

SP03.04 Students should conduct a variety of probability experiments, record the outcomes and discuss the results. For example, students could turn on a classroom radio. Note whether the first voice heard is a female or male voice. Students could then turn to a different station and record the gender of the voice. This could be repeated ten times. Students could then be asked to describe the probability of hearing a male voice as either impossible, certain, or possible.

SCO SP04 Students will be expected to compare the likelihood of two possible outcomes occurring, using words such as **less likely**, **equally likely**, or **more likely**. [C, CN, PS, R]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

SP04.01 Identify outcomes from a given probability experiment that are less likely, equally likely, or more likely to occur than other outcomes.
SP04.02 Design and conduct a probability experiment in which one outcome is less likely to occur than the other outcome.
SP04.03 Design and conduct a probability experiment in which one outcome is equally likely to occur as the other outcome.
SP04.04 Design and conduct a probability experiment in which one outcome is more likely to occur than the other outcome.

Performance Indicator Background

SP04.01 Provide the following using overhead spinners. As a class discussion, ask students,

- Which spinner is most likely to spin a 2?
- Which spinner is less likely to spin a 2?
- Which spinner is equally likely to spin a 2 or 3?

Note: Students should design and conduct probability experiments with spinners, bags of cubes, and coins. Suggestions are provided below.

SP04.02 Using a variety of coloured multi-link cubes, ask students to identify the number of coloured cubes needed to produce an outcome that is less likely to occur. For example, they may place 15 red, 10 blue, and 5 green in a bag. They would then state a colour that is less likely to be drawn.

Provide students with blank spinners and have them design an experiment with an event that is less likely to occur. For example, they may design a spinner with eight sections, four of which are yellow, two of which are red, one of which is blue, and one of which is green. Possible events would be, the spinner is less likely to land on blue than red, or green than yellow, etc.

SP04.03 Using a variety of coloured multi-link cubes, ask students to identify the number of coloured cubes needed to produce an outcome that is equally likely to occur. For example, place 10 red, 10 blue, and 10 green in a bag. Ask students which colour is likely to be drawn and explain their thinking.

SP04.04 Using a variety of coloured multi-link cubes, ask students to identify the number of coloured cubes needed to produce an outcome that is more likely to occur. For example, place 15 red, 10 blue, and 5 green in a bag. Ask students which colour is most likely to be drawn.

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